4 Things Nobody Tells You About Online News: a Model for the New News Market

Melika Liporace^{*} Tilburg University

February 2024

Abstract:

Social media create a new type of incentives for news producers. Consumers share content, influence the visibility of articles and determine the advertisement revenues ensuing. I study the new incentives created by sharing and evaluate the potential quality of ad-funded online news. Producers rely on a subset of rational and unbiased consumers to spread news articles. The resulting news has low precision and ambiguous welfare effects. Producers' incentive to invest in news quality increases with the private knowledge of the topic; hence, when information is most needed, the generated news tends to be of lesser quality. Competition does not necessarily improve news quality – it does so only if the sharing network is *sufficiently dense*. While ad-funded online news occasionally helps consumers take better decisions, it creates welfare mostly through entertainment. Some interventions, such as flagging wrong articles, substantially improve the outcome; other approaches, such as quality certification, do not.

JEL codes: D80, D85, L82, L11, L14

Keywords: Online News, News Quality, Social Media, Competition, Networks, Fact-Checking

^{*}This paper previously circulated as "The New News Market: a Model with Social Networks and Competition". I am grateful to Fernando Vega-Redondo, Andrea Mattozzi, Nenad Kos, Massimo Morelli, Ruben Durante, Gregorio Curello, Nadia Burani, Sašo Polanec and Andrea Pasqualini for their insightful comments. I further thank discussants and participants at Bocconi University internal seminars, the 2021 ES European Summer Meetings, the 35th EEA Congress, the 14th RGS Doctoral Conference, the 2021 MPI PolEc Summer School, the 19th EEFS Conference, the 14th PEJ Meeting, the XXXV JEI doctoral meetings, the 13th VPDE PhD Workshop, the 9th Warwick Economic PhD Conference and the 9th EBR & SEB LU Doctoral Conference. I gratefully acknowledge financial support from *Fondazione Invernizzi*. Any mistake is my own.

1 Introduction

The media landscape has evolved throughout history. From the press to radio, television and the rise of the internet age, many past revolutions gave rise to concerns about news quality. Nowadays, social media are under the spotlight. The idea that the online news market may be worse than traditional media is puzzling as it arises in a highly competitive environment. Yet, in the last decade, the rise in competition was accompanied by a decrease in media trust (see e.g. survey from Gallup [2020]). Understanding the effect of social media on the provision of information is important as the prevalence of online news is growing; the majority of American and European adults include online outlets to their media diet (Pew Research Center [2021], Pew Research Center [2018]). Observers increasingly fear market segmentation: this could result in a two-tier market where only those paying for articles would be well-informed. Is there hope for the ad-funded outlets to provide quality news, so that even free articles are informative? Should competition be encouraged or has social media metamorphosed the news market in a way that makes standard theory inapplicable?

While advertisement revenues and producers' reduced cost of entry date back more than a century, online outlets brought something new: sharing. With social media, consumers play an active role in spreading news article, raising their visibility, thereby producers' advertisement revenues. Hence, news producers behind ad-funded online outlets respond to new incentives. Because of advertisement revenues, articles now need to be shared online. In this sense, the very presence of a news sharing network changes the effects of the previously existing market environment.

In this paper, I evaluate the performance of such ad-funded online news outlets, focusing on the incentives linked to sharing behaviors. Three dimensions of the market environment are explored: the amount of private knowledge, the connectivity of the communication network and the presence of competition. After studying the effects of the environment on the provision of information, I question whether such outlets are welfare enhancing and propose possible interventions. Such questions are important to answer since the welfare consequences of the new incentives created by sharing have not yet been explored. This work thus offers new normative recommendations on the efficacy of ad-funded online news outlets and competition.

To explore these questions, I propose a general setup to represent the online news market. The market is populated by consumers and producers. The agents are concerned with some state of the world, for instance, whether vaccines are effective or not. All consumers observe a private signal, e.g. whether a vaccinated friend has developed the illness. In addition, some consumers, called *seeds*, come across news articles about vaccination directly and can decide to share it on an exogenous network to other consumers, called *followers*. Seeds care about sharing true news; followers read articles that seeds share, they are not part of the strategic interaction.

An article is a signal whose realization is informative about the state of the world. Given seeds' sharing behavior, producers decide on the quality of their outlet, i.e. the precision of the signal they send. Producers choose neither the state of the world on which to report nor the news realization, only the probability for the realization to correspond to the state of the world. In other words, producers only choose how many journalists to hire for their outlets, not what these journalists report; the more journalists, the higher the likelihood of reporting the true efficacy of vaccines. Each producer publishes one article about the same underlying state of the world, vaccine efficacy in this example, and only cares about how many consumers view their article. While the number of seeds reading a producer's outlet is exogenous, the number of followers seeing their article is endogenous. When several producers co-exist in the market, they compete *through seeds* to reach other consumers, as each consumer is restricted to see only one article.

This model brings interesting insights. Even when consumers are fully Bayesian, the market fails to deliver precise news. Thus, incentives created by social media do not suffice to induce high quality online news, even in a market populated by rational and unbiased agents. The business model based on advertisement revenues is flawed both because of the way it shapes producers' investment and because of the role of seeds who imperfectly channel information. The market environment then has counter intuitive effects on news quality: a lack of private knowledge is not substituted for by more informative articles and the influence of competition on news quality is tied to the connectivity of the social network on which news are shared. Furthermore, the presence of news outlets has ambiguous welfare consequences that not all interventions can overcome.

These results rely on two key mechanisms. First, the producers' incentive to invest is determined by the difference between the value of a true and a false article. Private knowledge, connectivity and competition all affect the value of true articles differently than that of false news, thus inefficiently modulating the producers' response to the market environment. Second, the market is shaped by consumers' sharing decision, which is determined by their private knowledge. Consumers' private knowledge thus bounds news quality. Below, I discuss in more detail how these two mechanisms drive all four main results.

First, ad-funded online outlets tend to fail when information is most needed (Proposition 2). News quality is less valuable for a producer in an environment with low private knowledge – either because the consumers are not well-informed by their signals, or because the state of the world is *ex ante* very uncertain. As private signals get noisier, consumers struggle distinguishing true and false news, leading them to treat any news article very similarly: the value of a true article decreases while that of false information increases. As one state of the world becomes more likely, investing in news quality gets more attractive for producers, since the difference between the value of true and false information is greater when the most likely state of the world realizes. These lead producers' incentives to be misaligned with consumers' need.

Second, competition can be detrimental; its effect depends on the network connectivity (Theorem 1). For any market structure, high connectivity negatively affects news quality; but it does so less strongly if the market is competitive. A monopolist's incentive to invest vanishes as the network gets very dense: one single node sharing would reach almost all other consumers then. The monopolist can thus create false content and rely on a few seeds receiving an erroneous private signal to reach many followers. This intuition does not follow through in competitive markets. Producers cannot rely on these few seeds anymore; articles need to be sufficiently shared in order to survive in the network. In this sense, competition decreases the value of a false article and thus pushes the producers towards more investment.

However, competition entails a second opposite effect. Splitting the market might be detrimental to investment, since the cost of news quality does not depend on the size of the market served. By accessing less initial seeds, the producer cannot reach as many followers, even if their article was shared by all seeds reading it. In this sense, competition decreases the value of a true article. The relative strength of these forces depend on the network degree: as connectivity increases, producers have access to more and more followers while competition inside the network becomes more biting. Therefore, competition is detrimental below a connectivity threshold.

Third, the presence of ad-funded online news is not necessarily beneficial to consumers (Theorem 2). Any equilibrium is Pareto inefficient. To go beyond the Pareto criterion, I consider different aspects of consumers' welfare. *Entertainment* – the utility derived from sharing – increases with news quality. To capture the value of information, I introduce an additional action, a bet, in which consumers must match the true state of the world. Agents are brought to better decision by news outlets if their expected utility from betting increases after reading a news article.

Generally, the market fails to let seeds take better decisions. This does not rely on the presence of competition or the timing of the game; but on the channeling role of the seeds. Producers have no incentive to publish articles more precise than the consumers' private signals, since this precision suffices to be shared by all readers. Therefore, the news quality is bounded by the consumers' knowledge and, for symmetric priors, seeds are always as well off by trusting their private signal for the bet. Followers, however, might take better decisions if the market is competitive: as the network tends to filter out false articles, the articles they end up seeing might be more precise than their private signal. Still, their utility from betting is bounded by the quality of consumers' private information.

Studying the decision to enter this bet at a cost allows to analyze whether online news pushes consumers towards action. Unsurprisingly, there exists a range of costs for which online news indeed helps agents enter the bet when it is beneficial to do so. More surprisingly, under mild conditions, there also exists a range of costs for which news outlets are detrimental to the consumers' capacity to enter the bet. Here too, the existence such of entry costs relates to the bound placed by consumers' private knowledge on news quality. By creating noise to consumers' private signals, online articles more often dissuade consumers from a beneficial bet than they prevent consumers from taking an unfavorable bet. When taking the price of the bet into account, agents are better off opting out of the risky action; yet, beneficial actions are less likely to be taken. Online outlets are thus particularly problematic when they cover topics which relate to action with positive externalities.

Fourth, I analyze the effects of fact checking (Proposition 6). I distinguish between *flagging*, i.e. fact checking articles; and *quality certification*, i.e. fact checking outlets. The former has substantial effects on welfare by removing the bound placed on news quality; the latter might marginally improve the news quality but does not remove the bound from private knowledge, hence it does not significantly affect welfare.

Flagging reduces the value of producing a false article by improving the seeds' filtering abilities. Interestingly, competition dilutes the effect of flagging. Actually, for any environment, there exists a level of flagging that makes competition detrimental. Indeed, flagging, like competition, reduces the value of false information; however, unlike competition, it does not decrease the value of true information. Flagging can then be seen as a substitute for competition: any outcome from competition is reproducible in uncompetitive markets through flagging. Certifying news outlets' quality allows producers to internalize the effects of their investment on the seeds' sharing strategy; however, the best outcome for producers is still to be shared all the time, which happens when they match consumers' private knowledge. Therefore, news quality is still bounded by the consumers' private knowledge.

The paper is organized as follows. Related literature is discussed in the remainder of this section. The general model is presented in Section 2. Section 3 analyzes the equilibrium resulting from a monopoly and a duopoly respectively; in particular, it assesses the role of the market environment on the outcome. Section 4 proposes a framework to assess welfare and analyzes it accordingly. Section 5 evaluates the effect of fact checking. Section 6 discusses the robustness of results and Section 7 concludes. The appendix presents the proofs omitted in the main text, as well as further extensions.

Related literature

This work offers two main contributions. To the best of my knowledge, this is the first paper to study the welfare consequences of the new incentives generated by online sharing and adrevenues. The analysis not only allows to propose a normative evaluation of a wide-spread business model, but also helps providing policy recommendations. Second, this paper is the first to study the effects of competition between news providers in a connected world. Doing so, I uncover a new mechanism through which competition in the news market might be detrimental.

More specifically, I contribute to several strands of the literature. I particularly relate to **theoretical** works on news markets, media economics and the spread of news in networks.

First, as to **unconnected news markets**, the existing theoretical literature accounts for the existence of low quality news in a competitive but unconnected world. Allcott and Gentzkow [2017] finds that uninformative news can survive if news quality is costly and if consumers cannot perfectly infer accuracy or if they enjoy partisan news. My setup is similar in that quality is costly and consumers cannot perfectly distinguish true from false articles. However, since it abstracts from reputation concerns, my mechanism does not fundamentally rely on outlets' quality being hidden; in addition, it does not require partisanship. In unconnected news markets, the ambiguous effects of competition between news providers has been widely explored. Namely, Gentzkow and Shapiro [2008] finds that competition is effective at reducing supply-driven biases, while its effects with demand-driven biases are ambiguous. Consistently with this conclusion, other authors find that competition has ambiguous effects when news consumers lack sophistication. For instance, Levy et al. [2022] studies how media companies can exploit consumers' correlation neglect. They find that competition reduces the producers'

ability to bias readers' beliefs, but that diversity has a cost in terms of optimal consumers' responses. Hu and Li [2018] and Perego and Yuksel [2022] study how rational inattention biases the provision of political information. Both find that competition inflates disagreement. Chen and Suen [2023] also finds that competition is detrimental to the accuracy and clarity of news when readers endogenously allocate attention between outlets whose editors are biased. This work significantly differs from the aforementioned analyses in that the detrimental effect of competition is not motivated by biases of either side of the market.

Second, as to **media economics**, this paper relates in particular to the influence of digitalization on media. Representative of this literature are the following papers. Anderson [2012] combines empirical and theoretical insights to offer an overview of the ad-financed business model in the internet age. Wilbur [2015] documents trends following digitalization for the mass media and how their business models has evolved. Finally, Peitz and Reisinger [2015] review various novel features resulting from new Internet media. I contribute to this literature by explicitly modeling one such new feature of online news market: shared content. I study its effects on producers incentives and equilibrium outcomes.

Note that Peitz and Reisinger [2015] briefly discuss how sharing decision might affect available content and link it to more general media biases. In this perspective, Hu [2021] studies the impact of media regulation in the digital age and finds that government regulation is rendered less effective by media biases inherent to the digital age. Because their model does not take into account any communication network, their analysis does not study interventions targeting the sharing behavior of consumers. My intervention evaluations, in contrast, only accounts for such incentives resulting from consumers' sharing decisions.

Third, as to **news in networks**, a connected world has rarely been the setup for news market models in the literature. To the best of my knowledge, only Kranton and McAdams [2022] study the effect of communication networks on the quality of information provided on the news market. While my model is inspired by the setup they propose, the questions I analyze significantly differ. Kranton and McAdams [2022] gives a compelling argument on how a network of consumers can change producers' investments. However, their mechanism abstracts from the role of competition. Because competition matters in determining both how detrimental is the network connectivity, and the efficacy of fact-checking, accounting for competition is essential for policy recommendations. Furthermore, Kranton and McAdams [2022] studies equilibrium news quality, but do not address the welfare effects of these market outcomes. Introducing welfare considerations to the analysis is an important contribution of this paper: it offers a normative evaluation of the new incentive generated by online sharing as well as policy implications which could not be derived otherwise.

Following the cascade literature,¹ Hsu et al. [2020] provides optimal conditions on a signal's precision for a cascade to occur when sharing is endogenous and strategic. This could, in turn, relate to a producer's objective, although no producer is featured in their setup. However, just as Kranton and McAdams [2022], Hsu et al. [2020] is set in an uncompetitive world. Finally,

¹This literature studies learning in networks when agents learn from actions. See Bikhchandani et al. [1992], Banerjee [1992] for their seminal work.

recent works explore the particular setup of learning on social media and possible reactions from the platforms. Bowen et al. [2021] study learning via shared news and find that polarization emerges when agents hold misconceptions about their friends' sharing behavior. They find that news aggregators help curb polarization. Papanastasiou [2020] studies the optimal fact-checking strategy of a social media platform. Contrarily to my setup, fact-checking is strategic in their analysis. However, some of their results still echo the present analysis; for instance, they find that, absent of fact-checking, sharing patterns are prone to the proliferation of fake news even if agents are only interested in sharing true news. None of these papers address the effects of competition between news providers in a connected world, or the welfare consequences of the incentive created by online sharing.

2 Model

2.1 Environment

The market is populated by news producers and news consumers. Consumers learn about an unknown state of the world $\omega \in \{0,1\}$ through news articles and private signals.² There is a common prior across all agents, $\Pr(\omega = 0) = w_0 \ge 1/2$. All agents are Bayesian.

I denote the (finite or infinite) set of news consumers I. All consumers receive an informative binary signal s about ω . These are i.i.d., with $\Pr(s = \omega | \omega) = \gamma$ for $\omega = 0, 1$. I further impose $\gamma > w_0$, so that consumers trust their private signal more than their prior.

In addition to private signals, the consumers come across news articles. Consumers who are directly exposed to news are called *seeds* and denoted i; otherwise, they are *followers* and denoted f. Followers see a news article if at least one neighbor shared. All consumers are exposed to at most one article, but some followers might be exposed to none – if no neighbors are seeds or when none of the neighboring seeds decided to share. When I do not want to explicitly distinguish seeds from followers, I denote news consumers j.

The consumers are arranged on a regular network of degree d.³ Consumers are randomly drawn to be seeds with probability b, so that the composition of one's neighborhood is random. News articles are assumed to travel on the network through seeds' shares. If several neighbors shared content from different sources, one of the article shared is picked at random – each share has the same probability to be seen. Therefore, the probability for a follower to see any given source is proportional to the number of neighbors sharing this source relative to the number of neighbors having shared any article. That is, the probability that f sees a given article k is:

 $Pr(f \text{ sees } k | A \text{ neighbors shared } k, B \text{ neighbors shared}) = \frac{A}{B}$

For instance, if 4 of f's neighbors shared a piece of information, but only 1 of them shared k, then, f sees k with probability 1/4, although f does see *some* article with probability 1.

 $^{^{2}}w$ denotes the outcome of ω . For the remainder of the paper, the distinction between random variables and their outcome is not made as long as it is clear from the context.

³In a regular network, all nodes have the same number of connections. Note this assumption is not fundamental to the analysis, but greatly simplifies the notation. The extension to non-regular networks is discussed in Section 6.

On the other hand, I consider a finite set of producers K.⁴ Each producer, denoted by k, publishes exactly one article.⁵ Each producer reaches a seed with the same exogenous probability $\frac{b}{K}$. The producer chooses the overall quality of the news that is published, but not the article's content. In other words, the producer sets the precision of the signal about the state of the world, not its realization.



Figure 1: Dark colored nodes are seeds; all articles are shared. Light colored nodes (followers) see a given producer with probability 1. Hatched nodes' (followers) source is determined at random – neutrally hatched nodes see a producer with probability 1/2, hatched nodes with a hue see the given producer with probability 2/3.

Figure 1 depicts the environment with two producers.

2.2 Timing, Equilibrium Concept and Objectives

All strategic interactions are assumed to be simultaneous.⁶ The only agents active in the game are initial seeds and producers. I focus on Nash Equilibria.

Seeds like sharing true information and dislike sharing false information; they are assumed to receive a positive payoff from sharing true information, a negative payoff when sharing false information, and payoff 0 if they do not share.⁷ In particular:⁸

$$u(\text{sharing article reporting } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -1 & \text{otherwise} \end{cases}$$

Seeds choose the probability with which they share an article. This can depend on the content that the article reports and whether it corresponds to the private signal they received. The article's content, i.e. the realization of the news signal, is denoted n; the congruence with the private signal is denoted S = +, -.⁹ The probability with which a seed shares an article from

 $^{^{4}}$ I abuse notation by denoting K both the set of producers and its cardinality

⁵Therefore, I can abuse notation by also denoting articles by k.

 $^{^6\}mathrm{The}$ sequential version of the game is discussed in Section 5

⁷This assumption can represent the interests of truth-seeking consumers. Implicitly, it also accounts for wider concerns such as reputation or attention. In Appendix B.2 seeds who seek attention for themselves have qualitatively similar best-responses.

 $^{^{8}}$ When asymmetric payoffs are considered, most results follow through, but additional equilibria might appear.

 $^{{}^{9}}S$ = + if the news content is the same as the seed's private signal, and S = – otherwise. The outcome of the

producer k whose content is n is denoted by $z_{S|n,k}$. Therefore, the seeds' strategy is a vector: $(z_{S|n,k})_{(S,n,k)\in\{+,-\}\times\{0,1\}\times K}$.

The producers choose the quality of their outlet to maximize their profits. The quality or precision of outlet k is defined as the probability of documenting the true state of the world, $q_k := \Pr(n = \omega | \omega)$ for $\omega = 0, 1$. Producers derive revenue from advertisement, hence from the visibility of their outlet. Their revenue is thus defined as the share of the network that sees their article.¹⁰ Their (total) cost is determined by cost function C, which is common to all producers. c denotes the marginal cost function. C is increasing and strictly convex, i.e. c(q) > 0 and c'(q) > 0. Without any investment in quality, the outlet produces uninformative news, that is, $q_k = 1/2$. Finally, c(1/2) = 0.

2.3 Best Responses

For ease of exposition, I derive the best-responses of initial seeds and producers for $w_0 = 1/2$. I then provide the best-response for general w_0 in a dedicated paragraph; details can be found in Appendix A.

2.3.1 Seeds' Problem

Take a seed who received private signal s and read a news article reporting n. Denoting p(n, s)i's posterior on the probability that the producer published a true article, the seeds expected utility from sharing is: p(n,s) + (1 - p(n,s))(-1) = 2p(n,s) - 1. Because a seed would share an article whose expected utility is positive, i shares content n upon receiving signal s when: $p(n,s) \ge 1/2$

A piece of news is true if it matches the state of the world, so that $p(n,s) \coloneqq Pr(\omega = n|n,s)$. Let seeds attribute prior probability q_k to an article from k being true. Using $w_0 = 1/2$ and Bayes' rule, we find:

$$p(0,0) = p(1,1) = \frac{\gamma q_k}{\gamma q_k + (1-\gamma)(1-q_k)} \quad \text{and} \quad p(0,1) = p(1,0) = \frac{(1-\gamma)q_k}{(1-\gamma)q_k + \gamma(1-q_k)}$$

As one would expect, all posteriors are increasing in q_k . Since no state of the world is ex-ante more likely, accounting for the possible (dis)agreement between private signal and news article is sufficient, and the subscript n can be omitted from the seeds' strategy. The identity of the news' producer being irrelevant to seeds beyond q_k , the subscript k is omitted as well. The seeds' best-response is summarized by $z = (z_+, z_-)$ and is characterized by:

private signal is $s \in \{0, 1\}$ while the congruence is $S \in \{+, -\}$. For instance s = 1 is a *positive* signal towards n being true if n = 1, and a negative signal towards the article being true if n = 0. If additionally $\omega = 1$, s = 1 and S = + are said to be *correct* while they would be *wrong* if $\omega = 0$.

 $^{^{10}}$ Intuitively, the revenues are scaled for size population because they relate to advertisement revenues. One might expect advertisers to be interested in the *portion* of the population a given news outlet is able to reach. Furthermore, with this representation, the model becomes scale-free, so that all results carry through with an infinite set of consumers. Finally, it allows profits to be bounded below 1

$$(z_{+}^{*}(q_{k}), z_{-}^{*}(q_{k})) = \begin{cases} (0,0) & \text{if } q_{k} < \underline{t} \\ (e,0) & \text{if } q_{k} = \underline{t} \\ (1,0) & \text{if } q_{k} \in (\underline{t}, \overline{t}) \\ (1,e) & \text{if } q_{k} = \overline{t} \\ (1,1) & \text{if } q_{k} > \overline{t} \end{cases}$$

for any $e \in [0, 1]$, where $\underline{t} = (1 - \gamma)$ and $\overline{t} = \gamma$.

Since $\underline{t} < \overline{t}$, the seeds' best response are weakly monotonic in q_k : $z_-^* \ge z_+^*$. In other words, one shares an article reporting the opposite of their private signal only if one would be ready to share this article, were it to report the same as their private signal. Hence, when q_k increases, the *ex-ante* probability for a node to share increases. Therefore, although the strategy z is multi-dimensional, the set of undominated z can be represented on a line.¹¹

Figure 2: Sharing Decisions of Seeds for Different News Quality

Figure 2 represents how sharing decisions is affected by different news quality, and the monotone aspect of it. Figure 3 displays seeds' best-response to news' quality q_k .



Figure 3: Best Response of Seeds as a Function of q_k

Best-response for general w_0

For $\gamma > w_0 > 1/2$, the news realization *n* matters in the beliefs that the article is true since a news reporting the most likely state of the world is more probable to be true: p(0,s) > p(1,-s). However, conditional on reading a given news content *n*, the seeds' best-response are as before:

$$(z_{+|n}^{*}(q_{k}), z_{-|n}^{*}(q_{k})) = \begin{cases} (0,0) & \text{if } q_{k} < \underline{t}_{n} \\ (e,0) & \text{if } q_{k} = \underline{t}_{n} \\ (1,0) & \text{if } q_{k} \in (\underline{t}_{n}, \overline{t}_{n}) \\ (1,e) & \text{if } q_{k} = \overline{t}_{n} \\ (1,1) & \text{if } q_{k} > \overline{t}_{n} \end{cases}$$

for any $e \in [0,1]$, where $\underline{t}_n = \frac{(1-\gamma)\Pr(\omega \neq n)}{(1-\gamma)\Pr(\omega \neq n) + \gamma\Pr(\omega = n)}$ and $\overline{t}_n = \frac{\gamma\Pr(\omega \neq n)}{\gamma\Pr(\omega \neq n) + (1-\gamma)\Pr(\omega = n)}$.

¹¹Formally, $z = (z_{+|0}, z_{+|1}, z_{-|0}, z_{-|1})$; for $w_0 = 1/2$, it is undominated to treat any news content the same way: $z_{+|0} = z_{+|1}, z_{-|0} = z_{-|1}$. For $q_k = \underline{t}$, any $z_{+|0} \neq z_{+|1}$ would also be undominated; however, setting $z_{+|0} = z_{+|1}$ leads to an equivalent analysis. The same applies to z_{-} for $q_k = \overline{t}$.

Because $\underline{t}_0 < \underline{t}_1 < \overline{t}_0 < \overline{t}_1$, the seeds' best response are again weakly monotonic in q_k ; the set of undominated strategies $z^* = (z^*_{+|0}, z^*_{+|1}, z^*_{-|0}, z^*_{-|1})$ can be represented on a line, as seen on Figure 4

Figure 4: Sharing Decisions of Seeds for Different News Quality

Notice that $z_{S|0}^* \ge z_{S|1}^*$: one shares an article reporting the least likely state of the world only if one would be ready to share this article, were it to report the most likely state of the world, given the same (dis)agreement with private signals.

2.3.2 Producers' Problem

Consider a producer k. Let R_k take value 1 if a consumer sees producer k's article. Assume that k is facing seeds who have strategy \mathbf{z} , while the other producers ℓ are investing \mathbf{q}_{ℓ} . Then, the expected profits for producer k who invests to reach quality q_k is:

$$\mathbb{E}(R_k|q_k;\mathbf{z},\mathbf{q}_\ell) - C(q_k)$$

The expected share of reader as a function of k's investment in quality is found as follows. For a random node to share the article from producer k, one needs: the consumer to be a seed – with probability b –, to come across k's article – with probability 1/K – and to share. Since $w_0 = 1/2$, the probability to share only depends on whether the news article corresponds to the private signal, which depends on whether k produced a true or false article. The probability for any consumer to share an article that reports a true/false content is thus:

$$p_T = \frac{b}{K}(\gamma z_+ + (1 - \gamma)z_-)$$
 and $p_F = \frac{b}{K}(\gamma z_- + (1 - \gamma)z_+)$

The *ex ante* probability that a consumer reads k's article represents the value of such article for producer k; it is denoted V_{X_k} , where $X_k = T, F.^{12}$ If producer k has no competitor, the probability to be read by publishing a news X is simply:

$$V_{X_k}(z) = \Pr(j \text{ seed}) + \Pr(j \text{ follower }) \Pr(\geq 1 \text{ neigh. shared}|X) = b + (1-b)(1-(1-p_{X_k})^d)$$

However, when producer k is not alone on the market, it is not enough that a follower's neighbor shared k's article; this follower also needs to see k against all other producers ℓ 's articles, which is influenced by the number of shares of ℓ 's articles. This depends on the content

 $^{^{12}}$ -X_k is the alternative, so X_k = T means -X_k = F and conversely

of other articles, which we denote $Y_{\ell} := (Y_l)_{l \neq k}$. Therefore:

$$V_{X_k}(z) = \Pr(j \text{ seed}) + \Pr(j \text{ follower}) \Pr(\geq 1 \text{ neigh. shared}) \Pr(j \text{ sees } k \text{ against } \ell)$$
$$= \sum_{Y_\ell} V_{X_k Y_\ell} \Pr(Y_\ell) = \frac{b}{K} + \sum_{Y_\ell} (1-b) \frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}} \left(1 - (1-p_{X_k} - p_{Y_\ell})^d\right) \Pr(Y_\ell)$$

where $V_{X_k Y_\ell}$ is the value for k of producing an article that is X = T, F when other producers have published articles which are true or false as described in Y_{ℓ} ;¹³ and denoting $p_{Y_{\ell}} = \sum_{l \neq k} p_{Y_l}$, with $Y_l = T, F$.

The probability for a follower to read information k given the other articles Y_{ℓ} has two factors. The former, $\frac{p_{X_k}}{p_{X_k}+p_{Y_\ell}}$ represents the expected share of followers k would get, conditional on them being reached by any news, whereas the latter factor $1 - (1 - p_{X_k} - p_{Y_\ell})^d$ represents the probability of any news to reach followers. It means that sharing affects the producer's revenue through two channels: the size of the total readership and the portion of readers viewing a given producer. The relative strength of these two effects depends on the connectivity of the network d. Both factors are however increasing in p_{X_k} . Hence, as long as true articles are shared more than false articles, true information is more visible, no matter the outcome of the competitor.

Finally, the expected portion of the network reached given an investment q_k is:

$$\mathbb{E}(R_k|q_k) = q_k V_T(z) + (1 - q_k) V_F(z)$$

Because the profits are $\mathbb{E}(R_k|q_k) - C(q_k)$, the maximization of profits implies:

$$q_k^*(z) = c^{-1} \big(V_T - V_F \big) \coloneqq c^{-1} \big(\Delta V_k(z; q_\ell) \big)$$

Because $c'(q) \ge 0$, the equilibrium investment $q^*(z)$ is (weakly) increasing in $\Delta V_k(z; q_\ell)$. Thus, $\Delta V_k(z;q_\ell)$ denotes producer k's incentive to invest. Intuitively, it corresponds to the additional number of views the producer gets in expectation from producing a true rather than a false article. Section 3 analyzes the shape of the function for one and two producers.

Best-response for general w_0

For $w_0 > 1/2$, in addition to the veracity of a news article, its realization n matters, as n = 0tends to be shared more $p_{X|0,k} \ge p_{X|1,k}$. In other words, the value of producing a X = T, F article also depends on the state of the world. The analysis of the producers' problem is however very similar:

$$\mathbb{E}(R_k|q_k) = w_0[q_k V_{T|0,k}(z) + (1-q_k)V_{F|1,k}(z)] + (1-w_0)[q_k V_{T|1,k}(z) + (1-q_k)V_{F|0,k}(z)]$$

where $V_{X|n,k}$ is the value of a X = T, F article reporting content n.¹⁴

 $[\]overline{\begin{smallmatrix} ^{13}\mathrm{Pr}(Y_{\ell}) = \prod_{l:Y_{l}=T} q_{l} \cdot \prod_{l:Y_{l}=F} (1-q_{l}); \text{ e.g. with two other producers } l_{1}, l_{2}, \mathrm{Pr}(T, F) = q_{1}(1-q_{2}).$ $\overset{^{14}\mathrm{Similarly}}{\overset{^{13}\mathrm{s}}{\mathrm{as above}}, \quad V_{X|n,k}(z) = \frac{b}{K} + \sum_{Y} (1-b) \frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}} \left(1 - (1-p_{X|n,k}-p_{Y|m,\ell})^{d}\right) \mathrm{Pr}(Y), \text{ where } m \coloneqq (m_{l})_{l \neq k} \text{ is defined implicitly by } Y \text{ given } X \text{ and } n, \text{ e.g. } X = T, n = 0 \text{ means } \omega = 0, \text{ so } Y_{l} = T \Leftrightarrow m_{l} = 0.$

Finally, the best-response is:

$$q_k^*(z) = c^{-1} \left(w_0 [V_{T|0,k} - V_{F|1,k}] + (1 - w_0) [V_{T|1,k} - V_{F|0,k}] \right) \coloneqq c^{-1} \left(\Delta V_k(z;q_\ell) \right)$$

Intuitively, $V_{T|0,k} - V_{F|1,k}$ corresponds to the additional number of views the producer gets in expectation from producing a true rather than a false article when the most likely state of the world realizes, $\omega = 0$; while $V_{T|1,k} - V_{F|0,k}$ correspond to the same concept for $\omega = 1$.

3 Equilibrium

In this section, I characterize possible equilibria in both a non-competitive and a competitive market. A market is said to be competitive when consumers see less articles than the amount available on the market, since in such case, producers are forced to compete through seeds in order to capture followers' views. Because I restrict consumers to receive only one piece of information, I analyze the outcome from a monopoly and a duopoly respectively. I study the equilibrium on each market with symmetric prior $w_0 = 1/2$. For the monopoly, I furthermore characterize the equilibrium and discuss the role of the environment for general w_0 .

3.1 Equilibrium without Competition

Consider a market with only one producer. For clarity purposes, I omit the k index in this section. The monopolist's incentive to invest is denoted $\Delta V_M(z)$ and can be written:

$$\Delta V_M(z) = (1-b) \left[(1-p_F)^d - (1-p_T)^d \right]$$

Let us now analyze the shape of such best-response:

Lemma 1. The monopolist's best-response to sharing $q^*(z)$ is single-peaked in z_+ , with maximum $\overline{z} \in (0;1]$; it is strictly decreasing in z_- . $q^*(z)$ is continuous in z.



Figure 5: Producer's Best Response

Figure 5 illustrates the shape of the producer's best response. Because the seeds' strategy is not a unidimensional object, I illustrate the shape of the producer's best-response on the set of seeds' undominated strategy. As before, I represent the seeds' strategy on a line and map the corresponding image as if the argument was unidimensional. The resulting function is non-monotonic. The hump shape is explained by the effect of the network. At first, when the probability for agents to share is low, every additional node sharing reaches an almost constant number of additional followers; because the probability that this share occurs after having issued a true article is higher, true information gains much more followers than false information – the best-response is increasing. But when *enough* shares would occur, any increase in the probability of sharing would lead to shares which are likely to reach followers that would have been reached anyways; the marginal value of the probability of sharing is decreasing, because of redundant path to followers in the network. Therefore, the number of followers reached with a false article, that is rarely shared, is increasing faster with z_+ than the number of followers reached with true news, making the best-response decreasing. On the segment with $z_- > 0$, agents start sharing news that does not correspond to their private signals. The probability that this concerns a false article is higher than the probability that it applies to true information, so that false information accumulates views faster than true news, making the best-response decreasing.

We can now characterize the Nash equilibrium of the monopoly.

Proposition 1. The equilibrium investment is unique, positive and leads to news quality $q_M^* = \min\{q^*(1,0), \bar{t}\}.$

Since any crossing occurs on the decreasing segment of $q^*(z)$, while $z^*(q)$ is weakly increasing,¹⁵ the intersection is unique. It lies either on the vertical part of $z^*(q)$, then $q^*(1,0) < \bar{t}$; or on the horizontal part of $z^*(q)$, then $q^*(1,0) \ge \bar{t}$.



Figure 6: Equilibrium with $q_M^* = q^*(1,0)$

Figure 7: Equilibrium with $q_M^* = \bar{t}$

Figure 6 shows the equilibrium with $q^*(1,0) < \bar{t}$. Figure 7 shows the equilibrium with $q^*(1,0) > \bar{t}$. Notice that Proposition 1 implies that $q_M^* < \max_z \{q^*(z)\}$. As in Kranton and McAdams [2022], the highest quality a producer could be willing to invest is not achieved.

¹⁵Technically, for $q_M^* = \overline{t}$, any $z_{-|0} \neq z_{-|1}$ is undominated, so there would exist equilibria with $z_{-|0} \neq z_{-|1}$. I abstract from this technicality as all results follow through.

Characterization for general w_0

The producer's best-response for general w_0 is very similar to that for $w_0 = 1/2$. In particular, with $\Delta V_M(z) = w_0 \left[(1 - p_{F|1})^d - (1 - p_{T|0})^d \right] + (1 - w_0) \left[(1 - p_{F|0})^d - (1 - p_{T|1})^d \right]$, the shape of the monopolist's best-response is as before given any realization of content n. The characterization of the equilibrium follows.

Corollary 1.

- The monopolist's best-response $q^*(z)$ is single-peaked in $z_{+|n}$, with local maxima $\bar{z_n} \in (0;1]$; it is strictly decreasing in $z_{-|n}$; $q^*(z)$ is continuous in z.
- There exists a unique Nash equilibrium. It features positive investment and is characterized by news quality $q_M^* = \max\{\min\{q^*(1,1,0,0), \bar{t}_0\}, \min\{q^*(1,1,1,0), \bar{t}_1\}\}.$

Appendix A provides figures illustrating producers' best responses and the equilibrium.

Below, I explore the role of the market environment on the equilibrium outcome, in particular, connectivity and private knowledge, through prior and precision of private signal.

3.1.1 The Role of Connectivity

High connectivity is generally detrimental to investment. In fact:

Lemma 2. For any sharing behavior z, the monopolist's incentive to invest is single-peaked in d; that is, there exists a threshold \bar{d} so that $\Delta V_M(z)$ is increasing for any $d < \bar{d}$ and decreasing for any $d > \bar{d}$.

In particular notice that as soon as $z_+ > 0$,¹⁶ $\Delta V_M(z;d) \to 0$ as $d \to \infty$. This means that as the network grows more connected, the producer's incentive to invest vanishes. This insight echoes Kranton and McAdams [2022]'s Proposition 3. To illustrate this point, consider a complete network, that is a network in which every consumer is connected with every other consumer. In such a context, a monopolist would need only a single share in order to reach every single consumers on the market; therefore, as long as one seed receives a different private signal than the others, the monopolist reaches the same number of consumers regardless on whether the article is true of false.

3.1.2 Role of Private Knowledge

Private knowledge encompasses two parameters: the prior about the state of the world, w_0 ; and the precision of private signals, γ . These represent respectively how *ex-ante* uncertain the documented topic is, and how well-informed agents privately are. Through different channels, both of these parameters have a similar effect on the producer's incentives.

¹⁶Technically, $z_{+|n} > 0$ for both n = 0, 1 is required.

Proposition 2. A decrease in private knowledge implies a decrease in the producer's incentive to invest: for any undominated z, $\Delta V_M(z)$ is increasing in both γ and w_0 .

Intuitively, when γ is low, consumers are not good at distinguishing true from false articles; hence, false information tends to be shared almost as often as true information. The value of a true article is low while that of a false article is high. Thus, the incentive for the producer to invest is low, as publishing a true article would not raise its visibility by a lot.

The channel through which *ex-ante* uncertainty affects the incentive to invest is different. Because seeds share more often articles whose content corresponds to the most likely state of the world, the difference of value between true and false news is greater when the most likely state of the world realizes. Hence, investment is more beneficial to the producer when $\omega = 0$. The expected profits from any given investment thus increases when the most beneficial state becomes more likely.

Proposition 2 underlines how the producers' incentives are at odds with consumers' interests. Therefore, one should not expect consumers, even fully rational consumers, to be able to discipline news production on this market. This result documents a fundamental shortcoming of the relatively new market structure of ad-funded online news outlets. It departs from sources of inefficiencies traditionally considered on news markets since it does not rely on any bias on any side of the market.

Note however that proposition 2 does not specify the effect of private knowledge on the equilibrium outcome. Indeed, γ and w_0 also affect the seeds' best-response. However, the equilibrium outcome is generally affected by a change in private knowledge in the same way as the producer's incentive to invest.

Corollary 2. Generally, a decrease in private knowledge implies a decrease in the equilibrium investment. In particular:

- (i) q_M^* increases with γ .
- (ii) q_M^* increases with w_0 if and only if $q_M^* \neq \overline{t}_0$.

Interestingly, a lack of private knowledge tends not to be compensated for by the market. Indeed, a decrease in private knowledge generally leads to worse information provision. Hence, the online news market fails exactly when it is the most needed: when the state of the world is very uncertain, either because of little prior knowledge, or because of poorly informative private signals. This indicates that the inefficiencies from online outlets deriving revenues from advertisement can be very problematic.

Another source of inefficiency appears from the market structure: the seeds' whose information is imperfect have a channelling role in this market.

Remark 1. In equilibrium, $q_M^* \leq \bar{t}_1$. Therefore, news quality is bounded by agent's private knowledge w_0 and γ .

A proper setup to formally study such inefficiencies is introduced in Section 4.

3.2 Equilibrium with Competition

I now assume that two producers coexist on the market. Because of tractability concerns, results are provided only for $w_0 = 1/2$. Let the two competitors be denoted by k and ℓ . The producer k's best response given ℓ 's investment and sharing strategy z can be rewritten:

$$\Delta V_k(z_k; z_\ell, q_\ell) = (1-b) \Big[V_T - V_F \Big] = (1-b) \Big[q_\ell \Big(V_{TT} - V_{FT} \Big) + (1-q_\ell) \Big(V_{TF} - V_{FF} \Big) \Big]$$

Where:

$$V_{X_k Y_\ell} = \frac{p_{X_k}}{p_{X_k} + p_{Y_\ell}} \Big(1 - (1 - p_{X_k} - p_{Y_\ell})^d \Big)$$

For any article published by ℓ , whether true or false $Y_{\ell} = T, F$, the visibility of k is higher for true articles than for false news, i.e. $V_{TY_{\ell}} \ge V_{FY_{\ell}}$. Therefore, $\Delta V_k(z_k; z_{\ell}, q_{\ell}) \ge 0$. In particular, the incentive to invest is strictly positive as long as true news is shared more often than false news, i.e. for any $z_{+|k} > z_{-|k}$; it is null for $z_{+|k} = z_{-|k}$.

The shape of $\Delta V_k(z_k; z_\ell, q_\ell)$ in z_k is similar to the monopoly case; however, k's best-response also depends on the sharing behavior of seeds reached be ℓ and ℓ 's investment.

Lemma 3. Duopolist k's best-response is as follows:

- (i) $q_k^*(z_k; z_\ell, q_\ell)$ is single-peaked in $z_{+|k}$ with maximum $\bar{z_k} \in (0; 1]$; it is strictly decreasing in $z_{-|k}$.
- (ii) $q_k^*(z_k; z_\ell, q_\ell)$ is decreasing in q_ℓ for $z_\ell = z_k$.
- (iii) $q_k^*(z_k; z_\ell, q_\ell)$ is continuous in z_k , z_ℓ and q_ℓ .

As before, the best-response of producer k is non-monotonic to his own seeds' sharing z_k . Interestingly, if the sharing pattern is the same among producers, their investments are strategic substitutes.

The NE can now be characterized. I call symmetric equilibria any equilibrium in which $z_k = z_\ell$ and $q_k = q_\ell$. In this case, $\Delta V_k = \Delta V_\ell$. I denote this common function $\Delta V_D(\mathbf{z}, q)$, and omit the producers' indices on the seeds' best response z.

Proposition 3. The symmetric equilibrium investment is unique, positive and leads to news quality $q_D^* = \arg \min_{q \in [1/2, \gamma]} |\Delta V_D((1, 0); q) - c(q)|.$

There are two cases to distinguish, as illustrated on Figure 8 and Figure 9.

When $c(\bar{t}) \geq \Delta V_D((1,0),\bar{t})$, as in Figure 8, there exists an intersection between c(q) and $\Delta V_D((1,0),q)$ in the interval $(1/2,\bar{t}]$ (left panel). Given that ℓ invests q_D^* , k's best response crosses the seeds' best response in $((1,0),q_D^*)$ (right panel). Therefore, this intersection is a NE.

When $c(\bar{t}) < \Delta V_D((1,0), \bar{t})$, as in in Figure 9, c(q) lies completely on the left of $\Delta V_D((1,0), q)$, without ever intersecting in the interval $q \in (1/2, \bar{t})$ (left panel). Equivalently, k's best response



Figure 9: Illustration of a case for which $q_D^* = \bar{t}$

 $c, \Delta V^D$

t

(0, 0)

 $(1,z_{-}^{D})$

(1,1)

(1,0)

to z given that ℓ invests $q_{\ell} < \bar{t}$ intersects the seeds' best response z above \bar{t} (right panel). This means that there does not exist a symmetric NE in which $z^* = (1,0)$. However, if $z_- > 0$, $\Delta V_D(z,q)$ is shifted to the left in the space $(\Delta V_D,q)$, so that it now crosses c(q) (left panel). Furthermore, because q_{ℓ} increases to \bar{t} , the curve $q_k^*(z,q_{\ell})$ is shifted downwards in the space (z,q_k) (right panel). For some $z_- > 0$, $\Delta V_D((1,z_-),\bar{t}) = c(\bar{t})$, so that $q_D^* = \bar{t}$.

While the symmetric equilibrium is unique, asymmetric equilibria generally exist and are not unique.

3.3 Effects of Competition

This subsection compares q_D^* and q_M^* . For cases to be comparable, $w_0 = 1/2$. I confront the two types of markets in terms of connectivity and signal precision. Since, $q_M^* \ge q_D^*$ only if $\Delta V_M(z) > \Delta V_D(z, q_D^*)$, I focus on comparative statics of $\Delta V_M(z)$ and $\Delta V_D(z, q_D^*)$.

3.3.1 The Role of Connectivity

The comparison between monopoly and duopoly depends on the connectivity of the network. Indeed, the presence of a competitor has an ambiguous effect on a producer's incentive to invest: on the one hand, investment might increase because each follower is harder to reach, so that the producer needs to be *sufficiently* shared; on the other hand, news quality might decrease because each producer reaches fewer seeds, so that fewer followers can be reached. In other words, by making the number of shares more important, competition decreases the value of false information; by reducing the producers' potential readership, it decreases the value of true information.

The strength of both of these forces depends on the connectivity. In a very connected network, each seed is connected to most followers, so that a producer *can* reach almost all followers even when a competitor is present. In a sparsely connected network, each follower is unlikely to be connected to several seeds, so that the probability to reach a follower is almost independent from the competitor's outcome. Therefore, competition should lead to lower investment in sparse network, but would be beneficial to news quality in dense networks. Theorem 1 formalizes this; in particular, there is a unique threshold for a network connectivity that determines which of the two forces dominate.

Theorem 1. There exists a unique threshold \bar{d} such that $q_M^*(d) \ge q_D^*(d)$ for all $d < \bar{d}$ and $q_M^*(d) \le q_D^*(d)$ for all $d > \bar{d}$

Proof. Define $DV(d) := \frac{\Delta V_M(z;d) - \Delta V_D(z,q;d)}{1-b}$. First, notice it that for d = 1, DV(d) > 0; however for $d \to \infty$, DV(d) < 0. Therefore, there must exist some d_0 such that $DV(d_0) \ge 0 > DV(d_0+1)$. All that is left to do is to show that such d_0 is unique. This is the case because if $DV(d_1) > DV(d_1+1)$ for some d_1 , then DV is decreasing for all subsequent $d > d_1$. See Appendix A for technical details.

Theorem 1 lends itself to both positive and normative interpretations. First, it underlines that, with news quality in mind, competition should only be encouraged in dense enough sharing networks. Second, it contrasts one of Kranton and McAdams [2022]'s central insight that very dense network are extremely detrimental to news quality. With competition, intermediate network density is still preferable;¹⁷ however, dense network are less detrimental to news quality in the presence of competition. In this sense, a highly competitive news market is more robust to high network connectivity, making very dense social media platforms less of a threat to news quality in the presence of competition.

To further illustrate the two mechanisms at play when competition is introduced, I consider a few specific instances. Take d = 1. Figure 10 depicts possible followers reached with one vs. two producer(s) in such a network with 12 nodes. On the left panel, the producer would reach four additional nodes, were their investment sufficient to make all seeds share, ; on the right panel, the same producer who now shares the market with a competitor can, at best, reach only two followers. Therefore, his incentive to invest when a competitor is present is half that of the case with no competition.

As the network's connectivity grows, this force vanishes, while the intensity of the competition increases. Figure 11 underlines how the strength of competition is made greater by a denser network. In the part of the network depicted, the central node happens to be a follower surrounded by seeds reading different articles. Most nodes reached by producer 1 (in red) do

¹⁷The duopolist's equilibrium investment in quality, $\Delta V(z)$ is hump-shaped in d; the insight and proof follows directly from Lemma 2.



Figure 10: Dark colored nodes are seeds.

not share. When d = 4, producer 1 still has probability 1/2 to reach the central node; with d = 8, his chances are only 1 out of 4 given the same sharing pattern.



Figure 11: Dark colored nodes are seeds who share; dark colored nodes with a grey circle are seeds who do not share. The probability of reaching the central follower is shown for d = 4 and d = 8.

This insight continues to apply as d grows. Take $d \to \infty$ such that that all nodes are connected to each other.¹⁸ All followers will be reached by an article as soon as the probability of sharing is not null, however arbitrarily small. For a monopolist, the incentive to invest vanishes, since only one seed sharing suffices to reach the entirety of the network. It is quite the opposite in a duopoly, since the probability of reaching a follower then depends on the total number of shares in the network.

3.3.2 The Role of Signal Precision

Remark 2.

- (i) When the seeds get perfectly informative private signal, monopoly yields higher investments than duopoly.
- (ii) When the seeds get perfectly uninformative private signal, the incentive to invest vanishes for both a monopoly and a duopoly.

¹⁸This requires $|I| \rightarrow \infty$, and assume that d grows as fast as |I|.

When the signal is perfectly informative, seeds only share true information. Then, the monopolist has the highest possible incentive to invest: false information is worthless; with true information, they reach all the followers the network allows them to reach. For the duopolist, false information is also worthless, but true information is less beneficial. Indeed, if the competitor released true information, they together reach the same portion of followers as the monopolist would have, but they split this audience in two; if the competitor released false information, the duopolist gets all followers reached, but reaches fewer followers than the monopolist would have since they are read by fewer seeds.

When the signal is perfectly uninformative, the result is very intuitive: the private signal is too noisy for agents to be able to tell true from false information; hence they treat both type of news without accounting for their private signal. Because the game is simultaneous, the producer does not internalize the effect of his investment on the consumers' prior, so that no investment is featured in equilibrium.

In general, private information is expected to have a stronger positive effect if the environment is not competitive. Intuitively, when the signal precision is high, consumers are relatively good at distinguishing true from false articles; therefore, very few seeds are sharing false news, and the monopolist cannot rely on them to reach enough followers. The positive effect of competition – harder to reach followers – is thus marginal; while its negative effect – reduced number of reachable followers – is still significant.

This insight is only shown with a numerical exercise. The example below details some numerical applications. For b and d high enough, there exists a threshold $\bar{\gamma}$ such that $\forall \gamma < \bar{\gamma}$, $\Delta V_D(\cdot; \gamma) > \Delta V_M(\cdot; \gamma)$. It indicates that competition is more likely to be beneficial in markets with little private information.

Example. Consider the sign of $\Delta V_M - \Delta V_D$ as a function of γ . I only consider the direct effects of γ on ΔV_D .¹⁹ In particular, I fix $q_{\ell}^* = 0.6$ and study k's best response. I assume $z^* = (1,0)$ for both producers. I report the threshold $\bar{\gamma}$ above which $\Delta V_M - \Delta V_D > 0$. I consider three level of broadcast reach and connectivity. The thresholds rounded to three digits are reported in Table 1.

	b = 0.25	b = 0.5	b = 0.75
<i>d</i> = 5	-	-	0.612
<i>d</i> = 10	_	0.820	0.914
d = 20	0.811	0.941	0.964

Table 1

Note that the thresholds reported are all above $q_D^* = 0.6$, which is consistent with $z^* = (1, 0)$. No threshold reported means that no matter the signal precision, monopoly creates a bigger incentive to invest than competition. Echoing Theorem 1, Table 1 illustrates how duopoly leads to a higher incentive to invest for a bigger range of signal precision as d and b increase.

¹⁹In a symmetric equilibrium, a change in γ indirectly influences ΔV_D through q_D^* . Considering indirect effects would require to specify a marginal cost function.

4 Welfare

So far, I have only been interested in the market outcomes, as measured by investment. Welfare has not been addressed. To start, I note that the market outcome is inefficient.

Proposition 4. Any equilibrium outcome on the news market with revenues derived from ads is Pareto inefficient

Proof. Take the case of a monopoly, with equilibrium $e^* = (q^*, z^*)$. Define $q^c(z; e^*)$ as the level of news quality that makes a consumer whose sharing decision is z indifferent between (q^c, z) and e^* . Likewise, define $q^p(z; e^*)$ as the level of news quality that ensures to the producer faced with sharing decision z the same revenue as e^* . If $\frac{\partial q^c}{\partial z} < \frac{\partial q^p}{\partial z}$, ²⁰ there is room for Pareto improvement since consumers require less investment to marginally increase their sharing than the producer is ready to offer for the same marginal increase in sharing. Now, the FOC of equilibrium imply $0 \le \frac{\partial q^c}{\partial z} < \infty$ while $\frac{\partial q^p}{\partial z} \to \infty$. The same reasoning applies to duopolists.

To analyze the welfare resulting from the production of news, I propose two approaches. The first one relates to the *entertainment* purpose of sharing behavior and is defined as the expected utility from sharing. In this sense, only seeds and producers are part of the analysis. However, entertainment does not capture how *informative* news are. It does not allow to judge whether, on average, agents take better decisions or are called to actions thanks to the information contained in articles from online news outlets. To address this question, I introduce an additional action to be taken by all news' consumers after the strategic interactions have unfolded. This measure of welfare also accounts for followers.

4.1 Framework of Analysis

Once the game is played out, all consumers can chose some action $a \in \{0, 1\}$ to match the state of the world. I think of this as a financial bet, but it can capture a wider range of utility derived from information. This action can depend on the private signal they receive and on the content of the article they read (if any). This bet has entry price r and consumers might decide to opt out of the bet after having observed their signal and the news article (if any). The decision to enter or opt out of the bet is denoted $e \in \{a, \emptyset\}$; opting out, they get an outside option of zero: $u(\emptyset) = 0$. The benefits from a match are the same as the loss from a mismatch:

$$u_j(a|\omega = w) = \begin{cases} 1 & \text{if } a = w \\ -1 & \text{otherwise} \end{cases}$$

From the consumer perspective, I study three different aspects of welfare: the utility derived

²⁰We abuse notation here in order to keep the intuition as clear as possible. While z is a vector, recall that, when q increases, the consumers would first share the most likely congruent news, then any congruent news, then the most likely news anyways, and then any news. Therefore, with ∂z , I mean to designate a marginal change in the sharing probability **in the relevant dimension**. So for instance if z=(1,0,0,0), ∂z is actually $\partial z_{+|1}$; if z = (0.5, 0, 0, 0), then I mean $\partial z_{+|0}$.

by seeds from sharing, the utility derived from betting and the capacity to enter the bet. I refer to the utility from sharing as $u_i(z)$, to the utility from betting as $u_i(a)$, and the capacity to enter the bet as $u_i(e)$. Note that the latter does not fully reflect the utility from entering the bet, which is $\mathbb{E}(\max\{u_i(a) - r; 0\})$.

The relevant expressions for each aspect of welfare are described in Appendix A. It is however important to note that each of these expressions do capture different elements related to welfare. First, despite the similarity in payoff structures and strategies, the expected utility from sharing and betting are different. This occurs because, if they do not believe the news content, seeds are constraint to not share but can still bet their private signal rather than the news. For instance, an outlet with $q_k = 0$, would lead to a null utility from sharing, but the maximal possible utility from betting.

Formally, the seeds' betting decision follows the same threshold rule as their sharing decision: they bet that the state of the world is the one reported in the article if the probability for the true state of the world to correspond to the news, p(n,s), is greater than 1/2. Therefore a(n,s) = nis played with probability $z_{S|n}$ ²¹ By a slight abuse of notation, I refer to this betting strategy as $z_{S|n}$ as well. The difference between $u_i(a)$ and $u_j(a)$ then lies in the way they vary with the news precision.

Remark 3. For any strategy with $z_{S|n,k} > 0$ for some (S, n, k), the expected utility from sharing is strictly increasing in q_k ; the expected utility from betting is not.

Notice that followers can bet according to a different strategy than seeds. In competitive markets, the precision of articles received by followers is higher than that of the outlets issuing them since the network filters out false articles. Followers' decision rule reflects this filtering.²²

Second, the utility from betting captures whether consumers more often make the right choice when taking a risky action whereas the capacity to enter the bet measures whether consumers more often take a risky action whose outcome would be positive. Both criteria therefore relate to the informativeness of the news, but from different perspectives. The former indicates whether information leads consumers to better choices, whereas the latter reflects whether news articles drive agents towards profitable actions.

Formally, after observing (n, s), consumers can chose to take or opt out of the bet. This decision depends on the price of the bet, r, and their expected benefit, which itself depends on the realization (n, s) and the precision of each signal. They opt out of the bet iff r > |2p(n, s) - 1|.

4.2Welfare for symmetric priors

Throughout this section, I assume $w_0 = 1/2$. I evaluate whether the presence of news outlets has welfare benefits for consumers and I discuss the effect of competition on total welfare.

²¹Formally, $\mathbb{E}(\mathbb{1}_{a=n|n,s}) = z_{S|n}$. When p(n,s) = 1/2, the seeds are indifferent between betting the news content or its opposite. The tie rule was chosen to keep consistency and does not influence the welfare analysis. ${}^{22}z_{S|n,k}^{f} = 1 \Leftrightarrow \sum_{m} \left[q_k \Pr(X, Y|\omega = n) \frac{p_{T|n,k}}{p_{T|n,k}+p_{Y|m,-k}} \Pr(\omega = n) - (1 - q_k) \Pr(X, -Y|\omega \neq n) \frac{p_{F|n,k}}{p_{F|n,k}+p_{Y|m,-k}} \Pr(\omega \neq n) \right] > 0$

Theorem 2. When no state of the world is ex ante more likely, the existence of news outlets has the following effects on consumers' welfare:

- (i) For seeds, the presence of news outlets does not improve their betting decision and is detrimental to their capacity to enter the bet for a non-zero measure set of r. It is beneficial to their expected utility from sharing.
- (ii) For followers, the presence of news outlets does not improve the betting decision and is detrimental to their capacity to enter the bet for a non-zero measure set of r if the market is not competitive or if the competitive symmetric investment $q_D^* \leq \frac{\gamma \sqrt{\gamma(1-\gamma)}}{2\gamma 1}$

As seeds cannot derive utility from sharing if there are no articles to share, the presence of news outlets has a positive effect on seeds' entertainment. Theorem 2 shows how consumers might be entertained but not informed by ad-funded online outlets.

With $w_0 = 1/2$, the news quality in any equilibrium is such that $q^* \leq \gamma$. Seeds are thus (weakly) better off betting their private signal. In other words, the presence of news outlets does not bring seeds to better decisions. Followers, however, might benefit from competition. In a competitive market, true news is more visible to followers since it is shared more often. Hence, articles seen by followers can be more precise than their private signal. However, the quality of news perceived by followers is still *bounded* by γ .

Lemma 4. In a competitive market, followers' utility from betting is bounded by the precision of their private signal: $\mathbb{E}(u_f(a)) \in [2\gamma - 1; \frac{3}{2}(2\gamma - 1)]$

Theorem 2 also shows how the presence of news outlets can be detrimental to the capacity to enter the bet. Intuitively, for low to moderate entry costs r, consumers would enter the bet without the presence of any news outlets, as their private signal is informative enough to justify the cost r. However, upon reading a news article whose content disagrees with their private signal, consumers are too uncertain about the state of the world to enter the bet. Now, because the news outlets are (weakly) more noisy than the private signals, consumers are more often wrongly than rightly dissuaded.

The intuition behind the proof is illustrated in Figure 12. For $r \in [\underline{r}, \overline{r}]$, seeds only participate to the bet if n = s. If, without an article, they would not have participated to the bet, the information transmitted thanks to news outlets is beneficial, as most seeds being prompted to participate are placing the right bet. However, if without an article, they would have participated to the bet, then the information transmitted thanks to news outlets is detrimental. Indeed, in such a case, the article is wrong more often than the private signal, so most seeds who opted out should have placed a bet.

It is important to underline that the presence of news outlets is not detrimental for the ex-ante utility of news consumers, $\mathbb{E}(\max\{u_j(a) - r; 0\})$. Because the bet is costly, taking the wrong decision by opting in is more costly than wrongly opting out of the bet. Therefore, at the individual level, consumers are not worse off with news outlets. Yet, risky actions that would be beneficial are taken less often because of the presence of the noise created by news outlets,



Figure 12: Illustration of Lemma 5's proof

which dissuades agents from taking a risky action. If such actions entail positive externalities, as would be for instance expected with vaccines, or if the perceived cost from the bet does not reflect its social cost, the imprecision from the news outlets would hurt consumers, even if they are fully Bayesian.

Finally, the negative effect of the presence of news outlets hinges on the cost of entering the bet. For moderate to high entry costs r, the presence of news outlets can be beneficial, as underlined in Lemma 5.

Lemma 5.

- (i) News' outlets are for seeds' capacity to enter the bet: beneficial for $r \in [r_s, \bar{r}]$, detrimental for $r \in [r, r_s)$, and neutral otherwise. These effects are strict for $q^* < \gamma$.
- (ii) For followers: the same applies for uncompetitive market; similar thresholds \underline{r}' and \overline{r}' exist if the market is competitive with symmetric investment $q_D^* < \frac{\gamma \sqrt{\gamma(1-\gamma)}}{2\gamma 1}$; and news outlets are never detrimental otherwise.

With
$$\underline{r} = 2 \frac{\gamma(1-q)}{\gamma(1-q) + (1-\gamma)q} - 1; r_s = 2\gamma - 1; \overline{r} = 2 \frac{\gamma q}{\gamma q + (1-\gamma)(1-q)} - 1$$

The same reasoning applies to followers. However, because of the filtering of news in competitive markets, the values of r that make followers change their behavior after having received a news article has to account for the quality of the news they perceive.²³

The welfare consequences of competition can now be assessed more carefully, by comparing consumers' expected gains from betting in a monopoly and a duopoly.

Proposition 5. Irrespective of the aspect of consumer welfare considered, competition can hinder total welfare even if $q_D^* > q_M^*$.

When considering the total welfare effect of competition, consumers and producers have to be considered. Seeds are not made better off and their quantity does not change in expectation. Followers might be better off if q_D^* is close enough to \bar{t} ; however, the number of followers encountering an article might be affected by competition. As seeds share more types of news, more

²³In particular, $\underline{r}' = 2 \frac{\sum_Y \gamma(1-q)V_{FY}}{\gamma(1-q)V_{FY} + (1-\gamma)qV_{T-Y}} - 1$ and $\overline{r}' = 2 \frac{\sum_Y \gamma qV_{TY}}{\gamma qV_{TY} + (1-\gamma)qV_{F-Y}} - 1$

followers come across possibly informative news, but the news also becomes less informative, as the network fails to filter out wrong articles. Producers split their readership while the total production cost doubles. Therefore, the effect of competition on welfare depends on the level of news quality, the connectivity of the network and the ratio of seeds in the population.

4.3 Welfare for asymmetric priors

When the prior about the state of the world w_0 is different than 1/2, the quality of the news is bounded by private *knowledge*. It means that the article published by news outlets might be more precise than private signals. However, generally, the insights from Section 4.2 are still applicable: the presence of news outlets have ambiguous effects on consumers' welfare.

Corollary 3. In uncompetitive markets, for any $w_0 < \gamma$:

- (i) Consumers are brought to better decisions iff $q^* > \gamma$.
- (ii) The gains from betting are still bounded by the precision of the private signal. In particular, $\mathbb{E}(u_j(a)) \in \left[2\gamma - 1; \frac{2\gamma - 1}{1 - 2\gamma(1 - \gamma)}\right]$

Corollary 4. In uncompetitive markets, for any $w_0 < \gamma$, and any $q_M^* < \max\left\{\gamma, \frac{w_0^2}{w_0^2 + (1-w_0)^2}\right\}$:

- (i) There exists a non-zero measure interval for r for which news outlets reduce consumers' capacity to enter the bet.
- (ii) The outlets effect on consumers' capacity to enter the bet can be non-monotonic in r.

5 Fact Checking

In this section, I study the effect of fact checking when applied to articles or to outlets. Applied to articles, I study how flagging false information affects welfare and its differential effect on news quality in non-competitive and competitive markets. Applied to news outlets, I question how much can quality certification improve producers investment.

5.1 Flagging

I wonder how flagging false information helps the provision of information on the market. In particular, let us assume that with some probability ρ , an information that does not correspond to the state of the world would be flagged by the platform on which seeds share before they decide whether to share. Because they care about truth only, such flagged information will never be shared. Hence, we can see flagging as perfectly informative signals, substituting the need for private signal. Therefore, one would expect this intervention to improve the outcome by decreasing the value of false information.

The producers' best responses can easily be rewritten to take into account the probability ρ that false information is flagged: for a monopolist $\Delta_M V(z, \rho) = V_T - (1 - \rho)V_F$; while for

duopolists who behave symmetrically $\Delta_D V(z,q;\rho) = qV_{TT} + (1-q)[(1-\rho)V_{TF} + \rho V_{T\varnothing}] - q(1-\rho)V_{FT} - (1-q)(1-\rho)[(1-\rho)V_{FF} + \rho V_{F\varnothing}]$, with $V_{X\varnothing}$ denoting the value of publishing a X = T, F article when the competitor has been flagged, that is, $V_{X\varnothing} = 1 - (1 - \frac{p_X}{2})^d$. Studying how $\Delta V_M(z;\rho) - \Delta V_D(z,q;\rho)$ evolves with ρ , I indeed find that flagging is more efficient in a monopoly.

Proposition 6.

- (i) Flagging has a stronger effect in monopolies than in duopolies.
- (ii) For any environment, there exists a level of flagging that makes competition detrimental;
 i.e. ∀(γ, b, d), ∃ρ': ΔV_M(z; ρ') ≥ ΔV_D(z, q^{*}_D; ρ'), where the inequality is strict for any positive probability of sharing, z₊ > 0.

Proposition 6 underlines how intervening is more difficult in competitive news markets. First, the same intervention has a stronger effect on a monopolist than on duopolists. Intuitively, competition dilutes the effect of flagging because of the strategic interaction between producers' investment. If flagging occurs more often, the value for any producer of publishing false information decreases; however, for a duopolist, it is more likely that the competitor has been flagged, and thus not to have to compete in the network in order to reach followers.

Recall that the effect of competition on news quality depends on the connectivity of the network because of the trade-off between followers who are harder to reach on the one hand; and the reduction of the potential readership on the other. Now, flagging false articles makes followers harder to reach anyways, with or without competition. Therefore, the benefits of competition are less and less relevant as flagging increases; the negative effect of competition however remains, since the maximum number of readers that can be reached is independent of the flagging probability.

The second result from Proposition 6 shows that competition is always detrimental to investements if flagging occurs often enough. Therefore, one can think of this intervention as a substitute for encouraging a change in the market structure towards more competitive markets. In fact, any market outcome from competition is reproducible though flagging.

Corollary 5. Any outcome $q_D^* > q_M^*$ is reproducible in a monopoly with some level of flagging ρ' ; i.e. $\exists \rho' : \Delta V_M(z; \rho') = \Delta V_D(z, q_D^*; 0)$.

Proposition 6 and Corollary 5 both use the following element: if all false articles are flagged, $\rho = 1$, monopoly yields higher incentive to invest than duopoly. This echoes Remark 2 as both $\rho = 1$ and $\gamma = 1$ make false information worthless. In fact, flagging can be seen as a substitute for consumers' private signal. Interestingly, flagging forces outlets to provide news that goes beyond consumers private knowledge, and thus to create informative content.

Remark 4. When false articles are flagged, news quality is not bounded by private knowledge anymore.

While these conclusions rely on a setup that ignores any type of partial partial partial distrust of the flagging institutions, they still underline the importance of flagging to counteract the weak incentives created by the business model of ad-funded online news outlets.

5.2 Quality Certification

I now wonder how welfare could be improved upon if the consumers were observing the actual quality of information. In terms of policy, this could for instance correspond to the role of a third party institution in charge of certifying the average quality of a news source, or an average fact checking score to be displayed on the online outlet.²⁴

Such a policy can be seen as having sequential moves.

In such a game, the seeds' best-responses would not change. Therefore, the threshold on news quality for which they would share any type of information, regardless of their private signal, does not change. This threshold is also the maximum achievable quality in a sequential game, which is set by the consumers' private knowledge.

Remark 5. Even when observable, news quality is bounded by private knowledge: $q^* < \bar{t}_1$. Therefore, the presence of news outlets still has ambiguous effects on consumers welfare.

Hence, most results from Section 4 apply when the quality of news outlets is observable.

Interestingly, both flagging and quality certification rely on the same type of policy: fact checking; yet, they have very different implications. This indicates that a major barrier to high quality online news is consumers' limited private knowledge; improving consumers' trust in news outlets is not sufficient to correct for the inefficiency generated by a business model in which revenues are generated by visibility.

6 Discussion

Most of the results exposed in this paper rely on the two following insights: the producers' incentive to invest is determined by the difference between the value of true and false articles and the consumers' private knowledge bounds news quality. These insights are robust to many extensions of the model.

6.1 Different setup

The setup studied so far analyzes a simplistic market in which all consumers are identical. All have the same number of neighbors, the same probability of being a seed, the same amount of private knowledge. These assumptions make my analysis more transparent but do not drive the main insights.

²⁴Such initiatives already exist, such as The Trust Project or Media Bias/Fact Check.

For instance, most results would directly apply if consumers were to have heterogenous signal precision. The probability to be shared given a X = T, F article would account for this heterogeneity; denoting $\psi(\gamma_i)$ the proportion of seeds with signal precision $\gamma_i, p_X = \frac{b}{K} \sum_i \psi(\gamma_i) (\gamma_i z_{i,S} + (1 - \gamma_i) z_{i,S})$ where S = + for X = T. The analysis would then be directly applicable. Note that the news quality in such a context would be bounded by the highest signal precision. Therefore, nodes whose signal precision is noisier would benefit from the presence of news outlets; results from Section 4 would then be mitigated to account for the distribution ψ . In particular, a low proportion of nodes with maximal signal precision would make the presence of news outlets more beneficial if the bound is reached; however, the bound would be less likely to be reached as the producers' could rely on the large proportion of nodes with noisier private signals to be shared.

Likewise, assuming irregular network or having heterogenous probability of being a seed would be inconsequential to the results as long as seeds are not targeted. The main text characterizes the expect benefit of a node given its degree and probability of being a seed. To account for heterogenity in this regard, it suffices to take the expectation of $\Delta V(\cdot; d, b)$ over the distribution of degrees or probability of being a seed. Further details are provided in Appendix B.

Interestingly, our analysis also permits to study the effect of the network structure, and in particular the degree distribution, on the investment in news quality. For instance, for network whose average degree is not too large, higher variation in degree would be detrimental to investment. This is due to the non-monotonic shape of $\Delta V(d)$ (see Lemma 2). However, if the degrees are very large, higher variance in the degree distribution is conductive to higher investment.

6.2 Different Objectives

Agents have straightforward and potentially simplistic objectives that offer tractability and clarity; however, one might wonder to which extend the results are robust to further considerations. In Appendix B, I explore how alternative or richer objectives could be incorporated. The main mechanisms are generally robust to such consideration, as briefly discussed below.

With a loss from sharing false news that is different than the benefits from sharing true news, more equilibria might exist; but the news quality is still bounded, and the bound is still a function of private knowledge. Furthermore, all results pertaining to producers' incentive are unaffected by the seeds' problem. In addition, the objective of seeds can be justified by other considerations than truth; for instance, seeds might be seeking *likes*. While the problem is more complex and less tractable, true information is still shared more often than false articles. Under mild conditions, private knowledge still bounds news quality; all results about producers' incentive to invest directly apply.

In the Appendix B.3, I also explore how consumers' biases might influence the market outcome and find they generally worsen news quality. For instance confirmation bias as modeled in Rabin and Schrag [1999] would increase the value of false information faster than that of true information, thus lowering the producers' incentive to invest. Likewise, a taste for sensationalism might make seeds less demanding: lesser news quality still makes them willing to share news, reducing the value of the upper bound placed on news quality. Finally, partian consumers, like confirmation bias, would make investment is less attractive from the point of view of the producers.

As for producers, one could argue that news quality matters beyond visibility. For instance, producers might get further benefits from being a reputable source of information. Generally, this would not affect the effect that the environment and competitiveness of the market have on producers' incentives, but the bound placed on news quality could be removed. These insights are detailed in Appendix B.3.

7 Conclusion

In this paper, I evaluate the performance of ad-funded online news outlets. I find that, without any intervention, they tend to be highly inefficient. First, news quality is bounded by the amount of private knowledge existing on the topic. Hence, the market does not compensate for a lack of private knowledge. High news quality is thus achievable only when the topic documented is already well-known: either because the outcome about this topic is rather certain; or because consumers are privately informed about it. The incentive created by sharing behaviors are a first cause for this result. Producers only care about being shared; as seeds rely on their knowledge to judge whether a content is worth sharing, having them share is not demanding when they are ill-informed. The second cause is the higher value of investment when the more likely state of the world realizes. Indeed, seeds are then more ready to share news documenting an expected state of the world. Thus, uncertain topics generate a lesser incentive to invest than topics for which information is less needed.

I additionally show that competition does not necessarily lead to better news quality. By comparing the outcomes of a monopoly and a duopoly, I conclude that monopoly is preferable in sparser networks populated by well-informed agents. This result puts into light two important forces appearing with competition. On the one hand, followers are harder to reach. This reduces the value of false information, as false articles would barely survive in the network when competing with true news. On the other hand, fewer followers can be reached. This reduces the value of true information, as an article shared by all seeds reading it would still reach few followers. When the network is sparse, the latter force dominates, making competition detrimental to news quality. This shows the limits of competition as a mean towards efficiency.

Furthermore, any online news market based on advertisement revenue is Pareto inefficient. I provide a framework to study welfare and find that online news outlets create value from entertainment but are not necessarily informative. In particular, the existence of online news rarely bring news consumers to take better decision; even when it does, their gain from it are still bounded by the precision of their private information. Furthermore, the presence of online news can be detrimental even when consumers are Bayesian, since it might discourage them from taking a risky action that would have actually been beneficial to them. A range of entry cost that makes online news detrimental to their capacity to take such action generally exist. Finally, I discuss how fact checking could improve news informativeness. Flagging false articles reduces their value, thus incentivizing producers to publish true articles more often. Because flagging substitutes private information, news quality is not bounded by private knowledge anymore. However, flagging is less efficient in competitive markets; actually, if false articles are flagged sufficiently often, competition is detrimental to news quality in any market environment. Therefore, one can substitute the positive effects of competition with flagging. To the contrary, allowing consumers to observe the quality of news outlets, for instance through a certification from an external institution, would not remove the bound placed on news quality by private knowledge.

This analysis is attractive because it gives consumers an endogenous control over information flow but not over news content. Furthermore, distortions that are inherent to a social network should be essential in underlining the differences between social media and other historical instances of ad-based business models for news. The central role of competition in this paper is reflected by its predominance in online outlets, as well as online networks. My analysis is robust to many extensions and puts into perspective the limits of the business model of ad-funded online news outlets; under such business models, the information provided online should not be expected reliable, even when all news consumers are rational and unbiased.

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A Proofs and computations

The Probability of Being Read by a Follower (as a Producer)

For any random vector $X \sim Multi(n,p)$ of dimension k, $X_1|_{X_1+X_2} \sim \mathcal{B}\left(n, \frac{p_1}{p_1+p_2}\right)$. Hence, $Pr(R_k = 1|\omega, n, m) = \frac{b}{K} + \frac{p_{X|n,k}}{p_{X|n,k}+p_{Y|m,\ell}} \left(1 - (1 - p_{X|n,k} - p_{Y|m,\ell})^d\right)$

Lemma 1: shape of monopolist best-response & interpretation

- Since $c^{-1}(\cdot)$ is increasing, $q^*(z)$ is increasing (resp. decreasing) in $z_{S|n}$ iff $\Delta_M V(z)$ is increasing (resp. decreasing) in $z_{S|n}$. Now:
 - Since any non-constant concave function f(x) defined on a closed interval is singlepeaked in x, it suffices to show that $\Delta V_M(z)$'s first derivative is decreasing in $z_{+|n}$. $\frac{\partial \Delta V_M(z)}{\partial z_{+|0}} \frac{1}{1-b} = -d(1-\gamma)(1-w_0)(1-b(1-\gamma)z_{+|0}))^{d-1} + dw_0\gamma(1-b\gamma z_{+|0})^{d-1}$ is decreasing in $z_{+|0}$: $\frac{1-b\gamma z_{+|0}}{1-b(1-\gamma)z_{+|0}} \leq 1$ decreases with $z_{+|0}$. The same holds for $z_{+|1}$.

$$- \frac{\partial \Delta V_M(z)}{\partial z_{-|0}} \frac{1}{1-b} < 0 \text{ since } \frac{w_0(1-\gamma)}{(1-w_0)\gamma} \left(\frac{1-b(\gamma+(1-\gamma)z_{-|0})}{1-b(\gamma z_{-|0}+1-\gamma)}\right)^{d-1} \le 1 \text{ The same holds for } z_{-|1-\beta|} < 0 \text{ since } \frac{w_0(1-\gamma)}{(1-w_0)\gamma} \left(\frac{1-b(\gamma+(1-\gamma)z_{-|0})}{1-b(\gamma z_{-|0}+1-\gamma)}\right)^{d-1} \le 1 \text{ The same holds for } z_{-|1-\beta|} < 0 \text{ since } \frac{w_0(1-\gamma)}{(1-w_0)\gamma} \left(\frac{1-b(\gamma+(1-\gamma)z_{-|0})}{1-b(\gamma+(1-\gamma)z_{-|0})}\right)^{d-1} \le 1 \text{ The same holds for } z_{-|1-\beta|} < 0 \text{ since } \frac{w_0(1-\gamma)}{(1-w_0)\gamma} \left(\frac{1-b(\gamma+(1-\gamma)z_{-|0})}{1-b(\gamma+(1-\gamma)z_{-|0})}\right)^{d-1} \le 1 \text{ The same holds for } z_{-|1-\beta|} < 0 \text{ since } \frac{w_0(1-\gamma)}{(1-w_0)\gamma} \left(\frac{1-b(\gamma+(1-\gamma)z_{-|0})}{1-b(\gamma+(1-\gamma)z_{-|0})}\right)^{d-1} \le 1 \text{ since } \frac{w_0(1-\gamma)}{1-b(\gamma+(1-\gamma)z_{-|0})} \le 0$$

• For any given $z_{S|n}$, $\Delta V(z)$ is a polynomial function of $z_{S|n}$, so it is continuous within each segment $z_{S|n} \in (0,1)$. The function is also continuous between segments. Indeed, $\lim_{z_{+|0} \to 1} \Delta V(z) = \lim_{z_{+|1} \to 0} \Delta V(z)$ and $\lim_{z_{+|1} \to 1} \Delta V(z) = \lim_{z_{-|0} \to 0} \Delta V(z)$.

In addition, note that the global maximum is $(1, \bar{z_1}, 0, 0)$ for some priors, and $(\bar{z_0}, 0, 0, 0)$ for all other priors. Indeed, for $w_0 = \gamma$, $\bar{q}_0 > \bar{q}_1$, while for $w_0 = 1/2$, $\bar{q}_0 < \bar{q}_1$.

Proposition 1 and Corollary 1: characterization of the monopoly equilibrium

First, we characterize the NE for $w_0 \neq \frac{1}{2}$. In any equilibrium $q \leq \bar{t}_1$ since $q = \bar{t}_1$ maximizes $\mathbb{E}(R_k|q_k)$. For $q = \frac{1}{2}, z_{+|0} = z_{+|1} = 1$. Hence, any equilibrium lies on the strictly decreasing segment of the producers' best response. Since $z^*(q)$ is weakly increasing, any intersection is unique. The intersection exists because both best responses are continuous.

The intersection is determined by the cost function. If $q^*(1,1,0,0) < \bar{t}_0$; it is easy to verify that $q_M^* = q^*(1,1,0,0)$ is an equilibrium. Likewise for $\bar{t}_0 < q^*(1,1,1,0) < \bar{t}_1$.

For $q^*(1,1,1,0) < \bar{t}_0 < q^*(1,1,0,0), q_M^* = \bar{t}_0$. Because $q^*(z)$ continuous, there exists some $z^*_{-|0}$ such that $c^{-1}(\Delta V_M(1,1,z^*_{-|0},0)) = \bar{t}_0$. The same applies for $\bar{t}_1 < q^*(1,1,1,0)$.

When $w_0 = 1/2$, $\bar{t}_0 = \bar{t}_1$, so that the characterization simplifies to $q_M^* = \min\{q^*(1,0), \bar{t}\}$.

Figure 13 and 14 illustrate two cases. Figure 6 shows the equilibrium with $q^*(1,1,0,0) > \bar{t}_0 > q^*(1,1,1,0)$. Figure 7 shows the equilibrium with $\bar{t}_1 > q^*(1,1,1,0) > \bar{t}_0$.



Lemma 2: the role of connectivity

 $\Delta V_M(z) \text{ is single-peaked in } d \text{ as it is the weighted sum of two single-peaked function of } d.$ $f(d) \coloneqq (1-p_{F|1})^d - (1-p_{T|0})^d \text{ is single-peaked because } f(d+1) - f(d) > 0 \text{ in } d = 0, f(d+1) - f(d) < 0$ for $d \to \infty$ and $f(d+1) - f(d) < 0 \Rightarrow f(d+2) - f(d+1) < 0$. Likewise for $(1-p_{F|0})^d - (1-p_{T|1})^d$.

Proposition 2 and Corollary 2: the effects of private knowledge

•
$$\frac{\partial \Delta V_M(z)}{\partial \gamma} \ge 0$$
 as $z_{+|0} - z_{-|1} \ge 0$ and $z_{+|1} - z_{-|0} \ge 0$.
• $\frac{\partial \Delta V_M(z)}{\partial w_0} \ge 0$ as $(1 - p_{F|1})^d - (1 - p_{F|0})^d \ge 0$ and $(1 - p_{T|1})^d - (1 - p_{T|0})^d \ge 0$.

Where the inequalities bind for z = (0, 0, 0, 0) and z = (1, 1, 1, 1).

Furthermore, γ and w_0 have the following effects on the seeds' best-response: $\frac{\partial \underline{t}_0}{\partial \gamma} < 0, \ \frac{\partial \underline{t}_1}{\partial \gamma} < 0, \ \frac{\partial \overline{t}_0}{\partial \gamma} > 0, \ \frac{\partial \overline{t}_1}{\partial \gamma} > 0$ and $\frac{\partial \underline{t}_0}{\partial w_0} < 0, \ \frac{\partial \overline{t}_0}{\partial w_0} > 0, \ \frac{\partial \overline{t}_1}{\partial w_0} > 0, \ \frac{\partial \overline{t}_1}{\partial w_0} > 0.$ Therefore, q_M^* unambiguously increases with γ . For w_0 , as $q^*(1,1,0,0)$ and $q^*(1,1,1,0)$ are weakly increasing in w_0 ; no increase in w_0 would change the inequalities detailed in Proposition 1. Therefore, q_M^* increases iff $q_M^* \neq \overline{t}_0$.

Lemma 3: shape of duopolist best-response

- (i) Since $c^{-1}(\cdot)$ is increasing, $q^*(z)$ is increasing (resp. decreasing) in $z_{S|k}$ iff $\Delta V(z)$ is increasing (resp. decreasing) in $z_{S|k}$. Now:
 - $-V_{T_kY_\ell}-V_{F_kY_\ell}$ is concave in $z_{+|k}$ for any $z_{+|k} \in [0,1], Y_\ell = T, F$. Indeed:

$$V_{T_kY_{\ell}} - V_{F_kY_{\ell}} = \frac{p_{T_k}}{p_{T_k} + p_{Y_{\ell}}} \left((1 - p_{F_k} - p_{Y_{\ell}})^d - (1 - p_{T_k} - p_{Y_{\ell}})^d \right) \\ + \left(\frac{p_{T_k}}{p_{T_k} + p_{Y_{\ell}}} - \frac{p_{F_k}}{p_{F_k} + p_{Y_{\ell}}} \right) \left(1 - (1 - p_{F_k} - p_{Y_{\ell}})^d \right)$$

 $\frac{p_{T_k}}{p_{T_k}+p_{Y_\ell}}$ and $(1-(1-p_{F_k}-p_{Y_\ell})^d)$ are both strictly increasing and weakly concave in $z_{+|k}$. In addition, $((1-p_{F_k}-p_{Y_\ell})^d-(1-p_{T_k}-p_{Y_\ell})^d)$ is single-peaked. As the product of weakly concave functions is weakly concave, all that is left to show is that

 $\frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}} \text{ is single peaked. Using } \frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}} = \frac{p_{Y_\ell}(p_{T_k} - p_{F_k})}{(p_{T_k} + p_{Y_\ell})(p_{F_k} + p_{Y_\ell})}, \text{ we find: } \frac{\partial \frac{p_{T_k}}{p_{T_k} + p_{Y_\ell}} - \frac{p_{F_k}}{p_{F_k} + p_{Y_\ell}}}{\partial z_{+|k}} = \frac{p_{Y_\ell} \frac{1}{2} b(2\gamma - 1) \left[p_{Y_\ell} - \frac{1}{4} b^2 \gamma(1 - \gamma) z_{+|k}^2 \right]}{2(p_{T_k} + p_{Y_\ell})^2 (p_{F_k} + p_{T_\ell})^2}, \text{ which is positive in } z_{+|k} = 0 \text{ and decreases with } z_{+|k}.$

$$- \text{ For } z_{-|k}, \text{ we have } \frac{\partial \Delta V(z_k; z_\ell, q_\ell)}{\partial z_{-|k}} = (1-b) \left(\Pr(Y_\ell) \frac{\partial V_{T_k Y_\ell} - V_{F_k Y_\ell}}{\partial z_{-|k}} \right) < 0 \text{ since } \frac{\partial V_{T_k Y_\ell} - V_{F_k Y_\ell}}{\partial z_{F_k}} \text{ is } \frac{1}{2} b \left[p_{Y_\ell} \left(\frac{(1-\gamma)}{(p_{T_k} + p_{Y_\ell})^2} \left(1 - (1-p_{T_k} - p_{Y_\ell}) \right)^d - \frac{\gamma}{(p_{F_k} + p_{Y_\ell})^2} \left(1 - (1-p_{F_k} - p_{Y_\ell})^d \right) \right) + d \frac{(1-\gamma)p_{T_k}}{p_{T_k} + p_{Y_\ell}} (1-p_{T_k} - p_{Y_\ell})^{d-1} - d \frac{\gamma p_{F_k}}{p_{F_k} + p_{Y_\ell}} (1-p_{F_k} - p_{Y_\ell})^{d-1}) \right], \text{ which is a sum of negative terms. Indeed, } the first term is negative because } \frac{1-(1-x)^d}{x^2} \text{ is decreasing in } x \text{ and } p_{T_k} \ge p_{F_k}. \text{ The second term is negative as } \frac{(1-\gamma)p_{T_k}}{p_{T_k} + p_{Y_\ell}} < \frac{\gamma p_{F_k}}{p_{F_k} + p_{Y_\ell}}.$$

- (ii) We prove that $(V_{TT} V_{FT}) (V_{TF} V_{FF}) \le 0$. Let us define $p_{X_k} = p_{X_\ell} := \frac{p_X}{2}$. Then we can rewrite $(V_{TT} V_{FT}) (V_{TF} V_{FF}) = -\frac{1}{2}(1 p_T)^d + (1 \frac{p_T + p_F}{2})^d \frac{1}{2}(1 p_F)^d$. Since $(1 x)^d$ is convex, $(1 \frac{1}{2}(x_0 + x_1))^d < \frac{1}{2}(1 x_0)^d + \frac{1}{2}(1 x_1)^d$.
- (iii) A potential source of discontinuity could arise with $z_{S|k} = z_{S|\ell} = 0 \forall S$; it is easy to verify $\Delta V_k(z_k; z_\ell, q_\ell)$ is defined and continuous in z = (0, 0, 0, 0). In addition, $\Delta V_k(z_k; z_\ell, q_\ell)$ is continuous between segments, as $\lim_{z_{+|k} \to 1} \Delta V(z_k; z_\ell, q_\ell) = \lim_{z_{-|k} \to 0} \Delta V(z_k; z_\ell, q_\ell)$.

Proposition 3: characterization of the duopoly symmetric equilibrium

First we characterize the symmetric equilibrium. Any equilibrium news quality lies in $[1/2, \bar{t}]$ since $\Delta V_D((1,1), q) = 0$. Now, consider two cases:

- 1. If $c(\bar{t}) \ge \Delta V_D((1,0),\bar{t})$, then $\exists \tilde{q} \in (1/2,\bar{t}]$: $c(\tilde{q}) = \Delta V((1,0),\tilde{q})$ since $c(1/2) < \Delta V_D((1,0),1/2)$, and c is increasing in q, ΔV_D is decreasing in q and both are continuous in q.
- 2. If $c(\bar{t}) < \Delta V_D((1,0),\bar{t})$, then $\exists \tilde{z}_F \in [0,1]$: $c(\bar{t}) = \Delta V((1,\tilde{z}_F),\bar{t})$ since $c(\bar{t}) > 0 = \Delta V_D((1,1);\bar{t})$ and $\Delta V_D(z;q)$ is continuous and decreasing in z_F .

The equilibrium is unique because any equilibrium lies on the part of $\Delta V_D(z;q)$ that is decreasing in z, and that c is increasing in q while ΔV_D is decreasing.

Theorem 1: shape of $\Delta V_M(z) - \Delta V_D(z,q)$ in d

Given $DV(d) := \frac{\Delta V_M(z;d) - \Delta V_D(z,q;d)}{1-b}$, we want to show that $DV(d) > DV(d+1) \Rightarrow DV(d+1) > DV(d+2)$. For readability, let us define for this proof: $c_1 = 1 - \frac{q}{2}$ $c_2 = \frac{1+q}{2}$ $c_3 = \frac{p_T}{p_T + p_F} - q$. Note that $c_1 > 0$, $c_2 > 0$ and c_3 's sign depends on z and q. In addition, we define: $A := c_1 \left((1 - p_T)^d p_T \right) - \frac{1}{2} c_3 \left(\left(1 - \frac{p_T + p_F}{2} \right)^d \frac{p_T + p_F}{2} \right)$ $B := c_2 \left((1 - p_F)^d p_F \right) + \frac{1}{2} c_3 \left(\left(1 - \frac{p_T + p_F}{2} \right)^d \frac{p_T + p_F}{2} \right) DV(d+1) - DV(d) < 0$
Then $DV(d) - DV(d+1) > 0 \Leftrightarrow B > A$. Notice that B > 0 because $c_3 > 0$ makes B a sum of positive term, and when $c_3 < 0$ A is a sum of positive term so that B > A > 0.

Likewise, $DV(d+2) - DV(d+1) = -(1-p_T)A + (1-p_F)B$. We can thus conclude $B > A \Rightarrow (1-p_F)B > (1-p_T)A$ since $p_T > p_F$ and B > 0.

Remark 2: the role of signal precision

- (i) When $\gamma \to 1$, the set of the seeds' best response reduces to $\{(1,0)\}$. Then: $\Delta V_M(z) = (1-b)(1-(1-b)^d) > (1-b)\left[\frac{q}{2}(1-(1-b)^d) + (1-q)(1-(1-\frac{1}{2}b)^d)\right] = \Delta V_D(z,q).$
- (ii) When $\gamma \to \frac{1}{2}$, $p_T = p_F$ for any z, so that the incentive to invest vanishes on both types of market: $\Delta V_M(z) = 0 = \Delta V_D(z;q)$

Section 4.1: Aspects of welfare

The relevant expected utilities are as follows:

- Seeds' expected utility from sharing is: $\mathbb{E}(u_i(z)) = \sum_{S,n,k} z_{S|n,k} [q_k \Pr(S, \omega = n) - (1 - q_k) \Pr(\neg S, \omega \neq n)] \frac{1}{K}$
- Seeds betting a(n,s) = n with probability $z_{S|n,k}$ have expected utility: $\mathbb{E}(u_i(a)) = \sum_{S,n,k} (2z_{S|n,k} - 1) \Big[q_k \Pr(S, \omega = n) - (1 - q_k) \Pr(\neg S, \omega \neq n) \Big] \frac{1}{K}$ Followers reading some news and betting n with probability $z_{S|n}^f$ have expected utility: $\sum_{m,S,n,k} \Big(2z_{S|n,k}^f - 1 \Big) \Big[q_k \frac{p_{T|n,k}}{p_{T|n,k} + p_{Y|m,-k}} \Pr(S, \omega = n, m) - (1 - q_k) \frac{p_{F|n,k}}{p_{F|n,k} + p_{Y|m,-k}} \Pr(\neg S, \omega \neq n, m) \Big]$ Where Y is implicitly determined by m and ω .²⁵
- Consumers' capacity to enter the bet is: $\mathbb{E}(u_j(e)) = \sum_{S,n,k} \mathbb{1}_{r < u_i(a|S,n,k)} u_i(a|S,n,k)$

Indeed:

- Conditional on receiving news n after private signal s, the utility from sharing is $\max\{2p(n,s)-1;0\} = z_{S|n,k}(2p(n,s)-1)$ since $2p(n,s)-1 > 0 \Rightarrow z_{S|n,k} > 0$. The expected utility from sharing is thus: $\sum_k \frac{1}{K} \sum_{s,n} z_{S|n,k}(2p(n,s)-1) \Pr(n,s)$. The final expression is fund by plugging the expression for p(n,s) and $\Pr(n,s)$ in the sum.
- Conditional on receiving news *n* after private signal *s*, accounting for the optimal decision to bet, the utility from sharing is $\max\{2p(n,s) - 1; 1 - 2p(n,s)\}$ As before, $2p(n,s) - 1 > 0 \Rightarrow z_{S|n,k} = 1$ Therefore, $\max\{2p(n,s) - 1; 1 - 2p(n,s)\} = (2z_{S|n,k} - 1)(2p(n,s) - 1)$. The final expression is found by plugging he expression for p(n,s) and $\Pr(n,s)$ into: $\sum_k \frac{1}{K} \sum_{s,n} (2z_{S|n,k} - 1)(2p(n,s) - 1) \Pr(n,s)$.

²⁵For instance, take $\omega = 0$, then m = 0 would lead to Y = T and -Y = F, while m = 1 would mean Y = F and -Y = T.

Without competition, the expression for followers' expected utility from betting is as that of seeds. Otherwise, $Pr(\omega = n|n, s, k) = \frac{q_k \Pr(T) \sum_Y \Pr(see \ T \text{ over } Y \) \Pr(\omega = n)}{\sum_w \Pr(n|\omega = w) \Pr(X) \sum_Y \Pr(see \ X \text{ over } Y \) \Pr(\omega = w)}$ and their utility is found as $|2Pr(\omega = n|n, s, k) - 1|$. Upon receiving no news, followers bet their private signal and get $2\gamma - 1$ in expectation.

• Upon each possible outcome (n, s), consumers do not enter if $u_j(a|n, s) < r$.

Theorem 2, Lemma 4 and Corollary 3: outlets' influence on betting decisions

Consider $w_0 = 1/2$.

- (i) The expected utility of a seed who would always bet the article's news is (2q − 1) ≤ 2γ − 1 ∀q ≤ γ. Seeds betting the news article only when n = s are bettings their private signal. Hence, seeds are always as well off following their private signal since in equilibrium q^{*} ≤ γ.
- (ii) Without competition, the expected utility of a follower receiving some article is the same as that of seeds. With competition, a follower's expected utility from betting *n* conditional on $n \neq s$ in any symmetric equilibrium is: $\mathbb{E}(u_j(a)|n \neq s) = \left[q^2(1-\gamma)\frac{1}{2} - (1-q)q\gamma\frac{p_F}{p_T+p_F}\right] + \left[q(1-q)(1-\gamma)\frac{p_T}{p_T+p_F} - (1-q)^2\gamma\frac{1}{2}\right]$, which is maximized, given any *q*, at z^* is such that $p_T = b\gamma$; $p_F = b(1-\gamma)$. Then $\mathbb{E}(u_j(a)|n \neq s) = \frac{(q^2(1-\gamma)-(1-q)^2\gamma)}{2}$. A follower is better off betting *n* rather than *s* when this is greater than 0, i.e. for $q \leq \frac{\gamma - \sqrt{\gamma(1-\gamma)}}{2\gamma-1}$.

A follower's maximal utility is found in $q = \gamma$, $z_k = z_{\ell} = (1, 0)$.

Consider any w_0 in an uncompetitive environment.

(i)
$$\sum_{w} \left(\sum_{s} \left(q \operatorname{Pr}(s=n) - (1-q) \operatorname{Pr}(s\neq n) \right) \operatorname{Pr}(\omega=w) \right) > 2\gamma - 1 \text{ iff } q > \gamma.$$

(ii) Consider $q^* \in (\gamma, \bar{t}_1]$. Then, $z_{+|0} = z_{+|1} = z_{-|0} = 1$. Then: $\mathbb{E}(u_i) \sum_{X,n} (2z_{S|n} - 1) [qPr(X) \Pr(\omega = n) - (1 - q)Pr(-X) \Pr(\omega \neq n)] = 2\gamma - 1 + 2(q(1 - \gamma)w_0 - (1 - q)\gamma(1 - w_0)) \le 2\gamma - 1 + 2(\bar{t}_1(1 - \gamma)w_0 - (1 - \bar{t}_1)\gamma(1 - w_0)) \le 2\gamma - 1 + \frac{2\gamma(1 - \gamma)}{\gamma^2 + (1 - \gamma)^2} (2\gamma - 1)$, with the last inequality from $\frac{\partial \frac{2w_0 - 1}{\gamma w_0 + (1 - \gamma)(1 - w_0)}}{\partial w_0} > 0$

Theorem 2, Lemma 5 and Corollary 4: outlets' influence on entering the bet

Consider $w_0 = 1/2$. We compare the decision to enter the bet with or without news.

- (i) For seeds: without news, they opt out of the bet for $r > r_s$ and enter the bet for $r \le r_s$. With news, seeds' behavior changes only in the interval $[\underline{\mathbf{r}}, \overline{\mathbf{r}}]$.
 - For r ∈ (r_s, r̄], agents with n = s place a bet, whereas they would not have without news. The probability for this bet to be won is γq, where with probability (1-γ)(1-q) it is lost. Since γq > (1 γ)(1 q), news are beneficial.

- For $r \in [\underline{r}, r_s]$, agents with $n \neq s$ opt out of the bet, whereas they would have entered it without news. The probability for seeds to benefit from this change is $(1-\gamma)q$; with $\gamma(1-q)$, they lose from the change as they would have won the bet. As $\gamma(1-q) > (1-\gamma)q$, news is detrimental to the decision to enter.
- (ii) Without competition or with a competitive symmetric investment $q_D^* \leq \frac{\gamma \sqrt{\gamma(1-\gamma)}}{2\gamma-1}$, the quality of the news received by followers $\leq \gamma$. Then, the reasoning in (i) applies with different threshold accounting for the filtering effect of the network.

Consider any w_0 in an uncompetitive environment. Seeds and followers receiving an article have the same expected utility from the decision to enter the bet. The decision to enter the bet might depend on the content of their private signal. Denote r_s be the bet price that makes consumers indifferent between betting or not, when $n = \emptyset$; $\underline{\mathbf{r}}_s$ that price for $n \neq s$; and that price for n = s. Then $\underline{\mathbf{r}}_s < r_s < \overline{r}_s$; and $r_0 > r_1$; $\underline{\mathbf{r}}_0 > \underline{\mathbf{r}}_1$; $\overline{r}_0 > \overline{r}_1$.

- (i) There are several possible relative order of the different thresholds.
 - If $q < w_0$, then $\bar{r}_1 < \underline{r}_0$. For $r \in [\underline{r}_0; r_0]$, the presence of news outlets only changes consumers' decision (from opt-in to opt-out) for $s = 0 \neq 1 = n$. The expected effect of news is: $-\gamma(1-q)w_0 + (1-\gamma)q(1-w_0) < 0$ since $q < \underline{t}_1$.
 - If $q \in \left[w_0; \frac{w_0^2}{w_0^2 + (1-w_0)^2}\right]$, then $r_1 < \underline{\mathbf{r}}_0 < \overline{r_1} < r_0$. For $r \in [\overline{r_1}; r_0]$, the presence of news outlets detrimentally dissuades consumers to enter the bet for $s = 0 \neq 1 = n$.
 - If $q > \frac{w_0^2}{w_0^2 + (1-w_0)^2}$, then $\underline{\mathbf{r}}_1 < \underline{\mathbf{r}}_0 < r_1 < r_0 < \overline{r}_1 < \overline{r}_0$. For $r \in [\underline{\mathbf{r}}_0; r_1]$, the presence of news outlets dissuades consumers to enter for any $s \neq n$. The expected effect of news is: $-\gamma(1-q) + (1-\gamma)q < 0$ for $q < \gamma$.
- (ii) Using the same reasoning, one notices that for $q < w_0$, the presence of news outlets is positive for $r \in [r_1, \bar{r}_1]$ and $r \in [r_0, \bar{r}_0]$ but negative for $r \in [\underline{r}_0; r_0]$, while $\bar{r}_1 < \underline{r}_0 < r_0$.

Proposition 5: effect of competition on total welfare

- For the expected utility from sharing: taking $d \to \infty$, profits decrease by $-2C(q_D^*)$ while expected sharing utility increases by $q_D^*\gamma - (1 - q_D^*)\gamma$. There exists a cost function C(q)such that $2C(q_D^*) > q_D^*\gamma - (1 - q_D^*)\gamma$, for instance $C(q) = \frac{q^2}{2}$.
- For the expected utility from betting: consider a cost function such that $q_M^* < q_D^* < \frac{\gamma \sqrt{\gamma(1-\gamma)}}{2\gamma 1}$. By Theorem 2, neither seeds nor followers are made better off by the presence of a second news outlet. Taking $d \to \infty$, the difference in total revenues from producers is the same with one or two producers, while the cost of production doubles.
- The same applies to the utility from entering the bet for $r < \underline{\mathbf{r}}$ and $r > \overline{r}$.

Proposition 6 and Corollary 5: effect of flagging with or without competition

Let the difference between incentives to invest with flagging be $FDV(z,q;\rho) \coloneqq \Delta V_M(z;\rho) - \Delta V_D(z,q;\rho)$. Then $\frac{\partial FDV(z,q;\rho)}{\partial \rho} = V_F + (1 - q)V_{TF} - (1 - q)V_{T\varnothing} - qV_{FT} - 2(1 - q)(1 - \rho)V_{FF} + (1 - q)(1 - 2\rho)V_{F\varnothing}$. This derivative is ≥ 0 , because $\frac{\partial^2 FDV(z,q;\rho)}{\partial \rho \partial q} \geq 0$, so that $\frac{\partial FDV(z,q;\rho)}{\partial \rho} \ge \frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2}$; and $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} > 0$.

Indeed, $\frac{\partial^2 FDV(z,q;\rho)}{\partial \rho \partial q} = \rho \left[1 + (1-p_F)^d - 2(1-\frac{p_F}{2})^d \right] + \left[(1-\frac{p_T+p_F}{2})^d - (1-\frac{p_T}{2})^d - (1-p_F)^d + (1-\frac{p_F}{2})^d \right]$ which is the sum of two positive terms since:

- the first term increases in p_F so that it is minimized in $p_F = 0$ where it is null.
- the second term increases in p_T so that it is minimum in $p_T = p_F = 0$ where it is null.

In addition, using $V_{FF} = \frac{1}{2}V_F$, one can rewrite $\frac{\partial FDV(z,q;\rho)}{\partial \rho}\Big|_{q=1/2} = \left[\frac{1+\rho}{2}V_F - \rho V_{F\varnothing}\right] + \frac{1}{2}\left[\frac{p_T - p_F}{p_T + p_F}\left(1 - \left(1 - \frac{p_T + p_F}{2}\right)^d\right)\right]$

- The first term is positive as $\frac{1+\rho}{2} > \rho$ and $V_F \ge V_{F\emptyset}$ $(V_F > V_{F\emptyset}$ for $p_F > 0)$.
- It is more cumbersome to show that the second term is positive. We show that it is non-decreasing in d and weakly positive for d = 1. We proceed by induction. To ease notation, define for the proof: $E(d) := \frac{p_T p_F}{p_T + p_F} \left(1 \left(1 \frac{p_T + p_F}{2}\right)^d\right) + \left(1 \frac{p_T}{2}\right)^d \left(1 \frac{p_F}{2}\right)^d$, $A := \frac{p_T}{2} \left[\left(1 \frac{p_T}{2}\right)^d \left(1 \frac{p_T + p_F}{2}\right)^d \right]$ and $B := \frac{p_F}{2} \left[\left(1 \frac{p_F}{2}\right)^d \left(1 \frac{p_T + p_F}{2}\right)^d \right]$. Then, E(d) E(d+1) = A B. Since $E(d+1) E(d+2) = \left(1 \frac{p_T}{2}\right)A \left(1 \frac{p_F}{2}\right)B$ and $A \ge 0$, the inductive step is straightforward and $E(d) E(d+1) \le 0 \Rightarrow E(d+1) E(d+2) \le 0$. Finally, it is easy to verify that for d = 1, E(1) E(2) = 0.

Therefore $\left. \frac{\partial FDV(z,q;\rho)}{\partial \rho} \right|_{q=1/2} > 0$ for any $p_F > 0$.

To show that there exists a level of flagging that makes competition detrimental to the producers' incentive to invest, it is enough to note that $\Delta V_M(z;\rho)$ is continuous in ρ and that with $\rho = 1$, $\Delta V_M(z;1) > \Delta V_D(z,q;1)$ To show that any outcome $q_D^* > q_M^*$ is reproducible in a monopoly, notice $\Delta V_M(z,q;1) > \Delta V_D(z,q;0)$.

B Extensions

B.1 Different Objectives: Asymmetric Loss From Sharing

In this section of the appendix, I consider a sharing payoff for seeds which accounts for possible asymmetries between the loss from sharing false news and the benefit from sharing true news. I then characterize the Nash equilibrium of this game and adapt the results of the main text to account for this asymmetry in payoff. I focus on results from Section 3 of the main text.

²⁶For A > 0 and $p_T > p_F$, $A \le B \Rightarrow (1 - \frac{p_T}{2})A < (1 - \frac{p_T}{2})B$: E(d) strictly increases for $d \ge 2$, $p_T > p_F$.

B.1.1 Best Response

Let the benefit be normalized the benefit to 1, and consider a loss λ when false news is shared. The seeds' payoff thus becomes:

$$u(\text{sharing article with content } n|\omega = w) = \begin{cases} 1 & \text{if } n = w \\ -\lambda & \text{otherwise} \end{cases}$$

The seeds' best-response is impacted in the following way: the thresholds according to which seeds start sharing different news content after their private signal reflects the asymmetry in payoff. We redefine:

$$\underline{t}_0^{\lambda} = \frac{\lambda(1-\gamma)(1-w_0)}{\lambda(1-\gamma)(1-w_0)+\gamma w_0} \qquad \overline{t}_0^{\lambda} = \frac{\lambda\gamma(1-w_0)}{\lambda\gamma(1-w_0)+(1-\gamma)w_0}$$
$$\underline{t}_1^{\lambda} = \frac{\lambda(1-\gamma)w_0}{\lambda(1-\gamma)w_0+\gamma(1-w_0)} \qquad \overline{t}_1^{\lambda} = \frac{\lambda\gamma w_0}{\lambda\gamma w_0+(1-\gamma)(1-w_0)}$$

Then, for $\gamma > w_0 > 1/2$, the seeds' best-response is exactly as before:

$$(z_{+|n}^{*}(q_{k}), z_{-|n}^{*}(q_{k})) = \begin{cases} (0,0) & \text{if } q_{k} < \underline{t}_{n} \\ (e,0) & \text{if } q_{k} = \underline{t}_{n} \\ (1,0) & \text{if } q_{k} \in (\underline{t}_{n}, \overline{t}_{n}) \\ (1,e) & \text{if } q_{k} = \overline{t}_{n} \\ (1,1) & \text{if } q_{k} > \overline{t}_{n} \end{cases}$$

for any $e \in [0,1]$, where $\underline{t}_n = \frac{(1-\gamma)\Pr(\omega \neq n)}{(1-\gamma)\Pr(\omega \neq n) + \gamma\Pr(\omega = n)}$ and $\overline{t}_n = \frac{\gamma\Pr(\omega \neq n)}{\gamma\Pr(\omega \neq n) + (1-\gamma)\Pr(\omega = n)}$.

The producers' best response does not change.

B.1.2 Equilibrium without Competition

Because the thresholds can now be all above or all below the no-investment quality 1/2, the bestresponses of seeds and producers might cross in many ways. Therefore, there might be multiple equilibrium investment and not all equilibria feature positive investment. Indeed, the seeds' best-response is not anchored around 1/2 anymore: it might lie completely above or below the producer's best-response; or might cross with the producer's best-response on non-monotonic segments. By considering all possible crossing given the shape of the respective best-responses, we can derive the conditions for positive investment and the maximal possible investment.

We define
$$\bar{q_0} \coloneqq q^*(\bar{z_{+|0}}, 0, 0, 0); \bar{q_1} = q^*(1, \bar{z_{+|1}}, 0, 0)$$
 and $\tilde{q_0} \coloneqq q^*(1, 0, 0, 0); \tilde{q_1} \coloneqq q^*(1, 1, 0, 0).$

Proposition B.1. 1. If either $1/2 \ge \bar{t}_1^{\lambda}$; or both $\bar{q}_0 < \underline{t}_0^{\lambda}$ and $\bar{q}_1 < \underline{t}_1^{\lambda}$, then there is a unique equilibrium with zero investment and $q_M^* = 1/2$. Otherwise, an equilibrium with positive investment exists; the highest equilibrium quality is:

- $q_M^* = \max\{\tilde{q_0}, \underline{t}_0^\lambda\}$ if $\bar{q_1} < \underline{t}_1^\lambda$,
- $q_M^* = \max{\{\tilde{q_1}, \underline{t}_1^{\lambda}\}}$ if $\underline{t}_1^{\lambda} \le \bar{x_1}$ and $\tilde{q_1} \le \bar{t}_0^{\lambda}$
- $q_M^* = \max\{\bar{t}_0^{\lambda}, \min\{q^*(1, 1, 1, 0), \bar{t}_1^{\lambda}\}\}$ otherwise.

Proof. First notice that any positive equilibrium investment has to lie within $[\underline{t}_0^{\lambda}, \overline{t}_1^{\lambda}]$. Indeed, it is easy to see that for any $q < \underline{t}_0^{\lambda}$, no news is ever shared so that the producer has no incentive to invest; likewise, $q = \overline{t}_1^{\lambda}$ is enough to insure that the producer's news is always share, so that any further investment does not yield any additional profits.

If $1/2 \ge \bar{t}_1^{\lambda}$, the producer's best response lies above the seeds' best response. No investment can be featured in equilibrium because consumers share all types of content without investment. If $\bar{q_0} < \underline{t}_0^{\lambda}$ and $\bar{q_1} < \underline{t}_1^{\lambda}$, the producer's best response lies below the seeds' best response. In such a case, there cannot be any investment either. Indeed, we know that $q < \underline{t}_0^{\lambda}$ cannot be an equilibrium. For any $q \in [\underline{t}_0^{\lambda}, \underline{t}_1^{\lambda}]$, by definition $\bar{q_0} = c^{-1}(\Delta V(z_{0,0}^-, 0, 0, 0)) \ge c(\Delta V(z^*(q)))$ so that $c(q) \ge c(\underline{t}_0^{\lambda}) > c(\bar{q_0}) \ge c(\Delta V(z^*(q)))$. Likewise, for $q \in [\underline{t}_1^{\lambda}, \overline{t}_0^{\lambda}]$, as $\bar{q_1} < \underline{t}_1^{\lambda}$, we have $c(q) \ge c(\underline{t}_1^{\lambda}) > c(\bar{q_1}) \ge c(\Delta V(z^*(q)))$.

If positive investment is possible, the investment has to be such that $q \in [\underline{t}_0^{\lambda}, \underline{t}_1^{\lambda})$. Because $\bar{q}_0 > \underline{t}_0^{\lambda}$, and $q^*(z)$ continuous, there there must exist some $z_{+|0}^*$ such that $c^{-1}(\Delta V(z_{+|0}^*, 0, 0, 0)) = \underline{t}_0^{\lambda}$. If $\tilde{q}_0 < \underline{t}_0^{\lambda}$, the maximal investment equilibrium is thus $\underline{t}_0^{\lambda}$; otherwise, \tilde{q}_0 is an equilibrium as $\tilde{q}_0 \in [\underline{t}_0^{\lambda}, \underline{t}_1^{\lambda})$ and by definition, $c(\tilde{q}_0) = \Delta V(1, 0, 0, 0)$, and leads to more investment. A similar reasoning applies to $\bar{q}_1 \ge \underline{t}_1^{\lambda}$ and $\tilde{q}_1 \le \overline{t}_0^{\lambda}$.

Finally, if $\bar{q_1} \ge \underline{t}_1^{\lambda}$ and $\tilde{q_1} > \bar{t}_0^{\lambda}$, because $q^*(z)$ is decreasing in $z_{-|0}$ and $z_{-|1}$, and continuous, there must exist a $q' \ge \bar{t}_0^{\lambda}$ and a $z' = (1, 1, z'_{-|0}, z'_{-|1})$ such that $c(q') = \Delta V(z')$. It is easy to verify that $\max\{\bar{t}_0^{\lambda}, \min\{q^*(1, 1, 1, 0), \bar{t}_1^{\lambda}\}\}$ yield the highest q on $[\underline{t}_0^{\lambda}, \underline{t}_1^{\lambda}]$ such that $c(q') = \Delta V(z')$. \Box

It follows that the bound on news quality is still a function of the quality of private information.

Remark B.1.1. In equilibrium, $q_M^* \leq \overline{t}_1^{\lambda}$. Therefore, news quality is still bounded by agent's private knowledge w_0 and γ .

The consequences on the comparative statistics are overall the same. In particular, all results pertaining to the effect of a parameter on the producer's incentive to invest can directly be applied as the producer's best-response is identical in this extension. Note the following change:

Corollary B.1.2. Take any increase in w_0 .

- For a marginal increase, the maximal equilibrium investment q_M^* increases iff $q_M^* \neq \underline{t}_0$ and $q_M^* \neq \overline{t}_0$
- For bigger increases, the maximal equilibrium investment q^{*}_M increases iff q^{*}_M ≠ t₀, q^{*}_M ≠ t₀ and c⁻¹ is steep enough, i.e. c⁻¹ is such that, for any w[']₀ > w₀, q^{*} > t₁ implies q^{*'} > t[']₁.

Proof. Under these respective conditions, the inequalities detailed in Proposition B.1.1 do not change. \Box

B.1.3 Equilibrium with Competition

As in the monopoly, new equilibria might appear when seeds' best-response depends on λ . Let $\bar{q}_k \coloneqq \max_{z_{+|k}} q_k^*((z_{+|k}, 0); 0)$ and $\tilde{q}_m = q^*((1, 0), (0, 0); 0)$

Define $\bar{z}_D \coloneqq \arg \max_{z_+} \left\{ \Delta V \left((z_+, 0), \underline{t}^{\lambda} \right) \right\}$ and $\bar{q}_D = \Delta V (\bar{z}_+^D, \underline{t}^{\lambda})$.

Proposition B.1.3. If $1/2 < \bar{t}^{\lambda}$ and $\underline{t}^{\lambda} \leq \bar{q}_D$, a symmetric equilibrium with positive investment exists; the highest equilibrium quality is $q_D^* = \arg \min_{q \in [t^{\lambda}, \bar{t}^{\lambda}]} |\Delta V_D((1,0);q) - c(q)|$.

Proof. Any equilibrium investment leads to a quality which lies in $[\underline{t}^{\lambda}, \overline{t}^{\lambda}]$. Indeed, since $\Delta V_D((0,0),q) = \Delta V_D((1,1),q) = 0$, for any $q < \min\{\underline{t}^{\lambda}, \overline{q}\}$ or $q > \max\{\overline{t}^{\lambda}, 1/2\}, c(q) > 0 = \Delta V_D(z^*(q),q)$.

Given $1/2 < \bar{t}^{\lambda}$ and $\underline{t}^{\lambda} \leq \bar{q}_D$, several cases can arise:

1. If $c(\underline{t}^{\lambda}) \leq \Delta V_D((1,0), \underline{t}^{\lambda})$ and $\Delta V_D((1,0), \overline{t}^{\lambda}) < c(\overline{t}^{\lambda})$, then $\exists \tilde{q} \in [\underline{t}^{\lambda}, \overline{t}^{\lambda}] : c(\tilde{q}) = \Delta V((1,0), \tilde{q})$, since c is increasing and $\Delta V_D((1,0), q)$ decreasing in q. Clearly, $(\tilde{q}, (1,0))$ is a NE.

It is the symmetric NE which leads to the highest investment. Indeed, assume there exists another symmetric equilibrium with investment $q' > q_D^*$. Then, $q' \in \{\underline{t}^{\lambda}, \overline{t}^{\lambda}\}$. For $q' = \overline{t}^{\lambda} > q_D^*$ to be part of an equilibrium, there must exist a $z' = (1, z'_{-})$ with $z'_{-} > 0$ such that $V_D(z', q') = c(\overline{t}^{\lambda})$. It is impossible, because $c(\overline{t}^{\lambda}) > c(q_D^*) = \Delta V_D((1,0), q_D^*) > \Delta V_D((1,0), \overline{t}^{\lambda}) > \Delta V_D((1,z_{-}), \overline{t}^{\lambda}) \quad \forall z_{-} > 0$, where the last inequality uses that $\Delta V_D(z;q)$ is decreasing in z_{-} .

- 2. If $c(\underline{t}^{\lambda}) > \Delta V_D((1,0), \underline{t}^{\lambda})$, then $\exists \tilde{z}_+ \in [\bar{z}_+^D, 1]$: $c(\underline{t}^{\lambda}) = \Delta V(\tilde{z}_+, \underline{t}^{\lambda})$, since, by assumption $\Delta V_D((\bar{z}_+^D, 0); \underline{t}^{\lambda}) > c(\underline{t}^{\lambda}) > \Delta V_D((1,0); \underline{t}^{\lambda})$, and $\Delta V_D(z;q)$ is continuous and decreasing on $[\bar{z}_+^D, 1]$. Clearly, $(\underline{t}^{\lambda}, (\tilde{z}_+, 0))$ is a NE.
- 3. If $c(\bar{t}^{\lambda}) < \Delta V_D((1,0), \bar{t}^{\lambda})$, then $\exists \tilde{z}_- \in [0,1]$: $c(\bar{t}^{\lambda}) = \Delta V_D((1, \tilde{z}_-), \underline{t}^{\lambda})$, since, by assumption $\Delta V_D((0,0); \bar{t}^{\lambda}) = 0 < c(\bar{t}^{\lambda}) < \Delta V_D((1,0); \underline{t}^{\lambda})$, and $\Delta V_D(z;q)$ is continuous and decreasing in z_- . Clearly, $(\bar{t}^{\lambda}, (1, \tilde{z}_-))$ is a NE.

Because the main text's comparison between monopoly and duopoly focuses on the producers' best-responses, all results follow through.

B.2 Different Objectives: Attention-Seeking Seeds

In this appendix, I explore an extension of the model with $w_0 = 1/2$ and symmetric behavior for all seeds' $z_k = z_\ell = z$. I assume that seeds do not intrinsically care whether the news they share is true or false; but they do care about receiving good feedback about it, e.g. a lot of *likes*. I characterize the best response of attention-seeking seeds to news quality q.

B.2.1 The Attention-Seeker Problem

I assume that seeds, contrary to producers, cannot observe the actual number of followers they reach; however, they can observe how many followers reacted to their shared post, as, typically, social media feature some sort of feedbacks, be it comments, likes, or re-shares. I focus on positive reactions, that I call *likes*, and assume that followers like a post if they receive a private signal consistent with it. As seeds, followers receive a binary signal that can agree or disagree with the news; however, they simply like a share if their private signal is congruent with the news – regardless of the prior probability for news to be true.

As before, seeds simultaneously choose whether to share the piece of news issued by ℓ , given their private signal s. Seeds decide to share if the amount of likes they expect to collect with their post exceeds a threshold $\tau \leq d$. τ can be interpreted as the value of an outside option – e.g. posting another type of article would yield τ likes – or, simply, the cost of sharing.

For consistency, I still denote R_{fi} the random variable which is one if f sees the post from i. As before, a follower sees only one post. If more than one neighbor shared a post, the follower sees the post from one random sharing neighbor, with uniform probability, that is:

$$Pr(R_{fi} = 1 | s \text{ neighbors of } f \text{ shared}) = \frac{1}{s}$$

where s is the outcome of the random variable S counting the number of f's neighbors who shared.

Define the random variable L_{fi} which is one if f likes the post shared by i. Recall that s is the random private signal that a follower receives. Then:

$$Pr(L_{fi} = 1) = Pr(L_{fi} = 1|R_{fi} = 1)Pr(R_{fi} = 1) = Pr(S = +)Pr(R_{fi} = 1)$$

A seed expects a different amount of likes for true and false information because, if read, true news get more likes than false information. The expected number of likes also depends on the visibility of the news, which in turn depends on the sharing decisions of all neighbors of each followers. Define n as the random variable counting the number of shares from f's neighbors, excluding i. The expected number of likes i gets from sharing a piece of information which is $X \in \{T, F\}$ is thus:

$$\mathbb{E}\left(\sum_{f\in\mathcal{N}_i} L_{fi} = 1 \middle| X\right) = dPr(f \text{ is a follower})Pr(S = +|X)\mathbb{E}\left(\frac{1}{P+1}\middle| X\right)$$

Now recall, upon reading a piece of news, seed i, too, gets a private signal about the truthfulness of the news, whose precision is γ . As before, all seeds have a common prior q_k about the probability for producer k to release true information. Let p(n,s) denote i's posterior upon receiving signal s and reading news n. Then, a seed decides to share a piece of information if and only if:

$$p(n,s)d(1-b)\gamma \mathbb{E}\left(\frac{1}{P+1}\Big|T\right) + (1-p(n,s))d(1-b)(1-\gamma)\mathbb{E}\left(\frac{1}{P+1}\Big|F\right) \ge \tau$$

Notice that the seeds' utility now depends on more than the producers' investment; it also depends on the behavior of other seeds. In particular, because seeds compete for likes, which occur only upon being seen, they would prefer a situation in which they are the only sharer. If true information is shared more, then this coordination concern would make them less prone to share true news; however, true information also brings more likes. Thus, there is a trade-off between visibility and veracity.

B.2.2 Seeds' Best Response

In this section, I focus on symmetric strategies $z_i = z \forall i$ and, by a slight misuse of language, I call best-response the pair of functions $(z_+^*(q), z_-^*(q))$ which maps q into [0,1] such that $z^*(q, \mathbf{z}^*(\mathbf{q})) = z^*(q)$.²⁷ Hence, given any investment q, I look at the subset of strategies which can be consistent with a symmetric equilibrium on the seeds' side.

As usual, p_X denotes the probability that a X = T, F news gets shared. Then, $n \sim \mathcal{B}(p_X, d-1)$ We can rewrite:

$$\mathbb{E}\left(\sum_{f\in\mathcal{N}_i} L_{fi} = 1|X\right) = d(1-b)Pr(S = +|X)\frac{1}{dp_X}\left(1 - (1-p_X)^d\right)$$

Thus, the expected number of like is:

$$p(n,s)\gamma \frac{1-b}{p_T} \left(1 - (1-p_T)^d\right) + (1-p(n,s))(1-\gamma) \frac{1-b}{p_F} \left(1 - (1-p_F)^d\right)$$

Lemma 6. For any $q, z_{+}^{*}(q) \ge z_{-}^{*}(q)$.

Proof. By contradiction, suppose that $z_F^*(q) > z_T^*(q)$, so that $p_F > p_T$. For this to be sustainable, we need $\mathbb{E}(\# \text{likes}|s_i = T) \le \tau \le \mathbb{E}(\# \text{likes}|s_i = F)$. However, this happens only when:

$$\frac{\gamma}{1-\gamma} < \frac{p_T}{(1-(1-p_T)^d)} \frac{(1-(1-p_F)^d)}{p_F}$$

Indeed, we have:

$$p_{\ell}(T;q_{\ell})\gamma \frac{1-b}{p_{T}} (1-(1-p_{T})^{d}) + (1-p_{\ell}(T;x_{\ell}))(1-\gamma) \frac{1-b}{p_{F}} (1-(1-p_{F})^{d})$$

$$< p_{\ell}(F;q_{\ell})\gamma \frac{1}{p_{T}} (1-(1-p_{T})^{d}) + (1-p_{\ell}(F;q_{\ell}))(1-\gamma) \frac{1}{p_{F}} (1-(1-p_{F})^{d})$$

²⁷Technically, each seed's best response would be a pair of (I + 1)-dimensional function, that each maps q and \mathbf{z}_{-i} into [0, 1], with I the random variable counting the number of seeds, and whose expectation is bI.

$$[p_{\ell}(T;x_{\ell}) - p_{\ell}(F;x_{\ell})]\gamma \frac{1}{p_{T}} (1 - (1 - p_{T})^{d}) < [p_{\ell}(T;q_{\ell}) - p_{\ell}(F;q_{\ell})](1 - \gamma) \frac{1}{p_{F}} (1 - (1 - p_{F})^{d})$$

$$\gamma \frac{1}{p_{T}} (1 - (1 - p_{T})^{d}) < (1 - \gamma) \frac{1}{p_{F}} (1 - (1 - p_{F})^{d})$$

Because $\frac{\gamma}{1-\gamma} > 1$, we need $\frac{p_T}{(1-(1-p_T)^d)} \frac{(1-(1-p_F)^d)}{p_F} > 1$. Now $f(x) = \frac{x}{(1-(1-x)^d)}$ is an increasing function of x; indeed, we have:

$$\operatorname{sign}\left(\frac{\partial f}{\partial x}\Big|_{x\in(0,1)}\right) = \operatorname{sign}\left(\frac{1-(1-x)^d - xd(1-x)^{d-1}}{(1-(1-x)^d)^2}\right) = \operatorname{sign}\left(1-(1-x)^d - xd(1-x)^{d-1}\right)$$

Now, $g(x) := (1-x)^d - xd(1-x)^{d-1} < 1$ over $x \in [0,1]$. Indeed, g(0) = 1, g(1) = 0, and g strictly decreasing in-between, since:

$$\frac{\partial g}{\partial x}\Big|_{x\in[0,1]} = (d-1)(1-x)^{d-2} \Big[(1-x) - (1+x(d-1)) \Big] = (d-1)(1-x)^{d-2} [-xd] \le 0$$

Since $\frac{1-(1-x)^d - xd(1-x)^{d-1}}{(1-(1-x)^d)^2} \ge 0$ on [0,1], *f* is indeed increasing on [0,1]

Therefore, we conclude that $p_T > p_F$, contradicting our initial hypothesis.

Corollary 6. $\mathbb{E}(\# likes | X)$ is increasing in p(n,s), for any $S \in \{+,-\}, X \in \{T,F\}$.

Proof. It is enough to notice that, since $p_T > p_F$: $\frac{p_T}{(1-(1-p_T)^d)} \frac{(1-(1-p_F)^d)}{p_F} > 1$ so that the coefficient of p(n,s) is positive.

I can now characterize the symmetric best-response of attention-seeking seeds

Proposition 7.

(i) For any $\tau \leq \gamma \delta$, $z_+^*(q; \tau) = z_-^*(q; \tau) = 1$ if and only if $q \geq \hat{q}(\tau)$.

(*ii*) For any $\tau \ge (1 - \gamma)d(1 - b)$, $z_{+}^{*}(q; \tau) = z_{-}^{*}(q; \tau) = 0$ if and only if $q \le q(\tau)$.

(*iii*) For any $\tau \in [\tau_1, \tau_2]$, $z_+^*(q; \tau) = 1$, $z_-^*(q; \tau) = 0$ if only if $q \in [q_1(\tau), q_2(\tau)]$.

Where:

$$\delta(b) = \frac{1-b}{b} [1 - (1-b)^d], \quad \tau_1(b) = \frac{1-b}{b} [1 - (1 - b(1-\gamma))^d], \quad \tau_2(b) = \frac{1-b}{b} [1 - (1 - b\gamma)^d]$$

And, given $Q = \frac{\frac{b\tau}{1-b} - 1 + (1-b(1-\gamma))^d}{(1-b(1-\gamma))^d - (1-b\gamma)^d}$,

$$\hat{q}(\tau) = \frac{\gamma}{2\gamma - 1} \frac{\tau - (1 - \gamma)\delta}{\tau}, \quad \hat{q}(\tau) = \frac{1 - \gamma}{2\gamma - 1} \frac{\tau - (1 - \gamma)d(1 - b)}{d(1 - b) - \tau}, \quad q_1(\tau) = \frac{(1 - \gamma)Q}{(1 - \gamma)Q + \gamma(1 - Q)}, \quad q_2(\tau) = \frac{\gamma Q}{\gamma Q + (1 - \gamma)(1 - Q)}$$

Proof. (i) Given $\tau \leq \gamma \delta$, if $q \geq \hat{q}(\tau)$, it is easy to verify that always sharing is a best response, i.e. $\mathbb{E}(\# \text{ likes } |T, \mathbf{z}_{-\mathbf{i}} = (\mathbf{1}, \mathbf{1})) > \mathbb{E}(\# \text{ likes } |F, \mathbf{z}_{-\mathbf{i}} = (\mathbf{1}, \mathbf{1})) \geq \tau$. Indeed, if every other seeds always share, $p_T = p_F = b$, then the expected number of likes upon receiving a false signal is:

$$[p(F;q)\gamma + (1-p(F,q))(1-\gamma)]\frac{1-b}{b}(1-(1-b)^d)$$

Which is higher than τ iff: $p(F;q) \geq \frac{\frac{\tau}{\delta(b)} - (1-\gamma)}{2\gamma - 1}$. Given that $p(F;q) = \frac{(1-\gamma)q}{(1-\gamma)q + \gamma q}$, this happens iff $q \geq \frac{\gamma}{2\gamma - 1} \frac{\tau - (1-\gamma)\delta}{\tau} = \hat{q}(\tau)$. Because $\tau \leq \gamma\delta$, $\hat{q}(\tau) \leq 1$; for $\tau < (1-\gamma)\delta$, $\hat{q}(\tau) < 0$, the condition is always fulfilled.

For proving the converse, recall that $\frac{1-(1-p)^d}{p}$ is decreasing in p. Suppose there exists another p' < b that is sustained in equilibrium. Then, $\mathbb{E}(\# \text{ likes } | F, p' < b) > \mathbb{E}(\# \text{ likes } | F, p = b) \ge \tau$ so that i would have an incentive to deviate towards $p_i = 1$.

(ii) Likewise, given $\tau \ge (1-\gamma)d(1-b)$, if $q \le q(\tau)$, then even $d(1-b)[p(T,q)\gamma+(1-p(T,q))(1-\gamma)]$ likes are not enough for anyone to share, so that (0,0) is a the best response to any p given q and τ . Indeed, if every other seeds never share, $p_T = p_F = 0$. Then, the expected number of likes upon receiving a true signal is:

$$[p(T;q)\gamma + (1 - p(T,q))(1 - \gamma)]d(1 - b)$$

Which is lower than τ iff: $p(T;q) \leq \frac{\frac{\tau}{d(1-b)} - (1-\rho)}{2\rho-1}$. Given that $p(T;q) = \frac{\gamma q}{\gamma q + (1-\gamma)q}$, this happens iff $q \leq \frac{1-\gamma}{2\gamma-1} \frac{\tau - (1-\gamma)d(1-b)}{d(1-b)-\tau} = \tilde{q}(\tau)$. Because $\tau \geq (1-\gamma)d(1-b), \ \tilde{q}(\tau) \geq 0$; for $\tau > \gamma d(1-b), \ \hat{q}(\tau) > 1$, the condition is always fulfilled.

(iii) Again, we can simply verify that, given $\tau \in [\tau_1, \tau_2]$, if $q \in [q_1(\tau), q_2(\tau)]$ and every -i seed is sharing only when they receive a positive signal, $\mathbb{E}(\# \text{ likes } | T) \ge \tau \ge \mathbb{E}(\# \text{ likes } | F)$. Any $z_{-i,-} > 0$ would lower the $\mathbb{E}(\# \text{ likes } | F)$ further away from τ , making i set $z_{i,-} = 0$; any $z_{-i,+} < 1$ would increase the $\mathbb{E}(\# \text{ likes } | T)$ further away from τ , making i set $z_{i,+} = 1$.

If every other seed shares only upon receiving a positive signal, $z_{-i,+} = 1, z_{-i,-} = 0$. Then, *i* also only shares upon receiving a positive signal iff:

$$p(T;q)\frac{1-(1-b\gamma)^d}{b} + (1-p(T,q))\frac{1-(1-b(1-\gamma))^d}{b}) > \tau > p(F;q)\frac{1-(1-b\gamma)^d}{b} + (1-p(F,q))\frac{1-(1-b(1-\gamma))^d}{b}$$

Which is possible only if $\tau \in [\tau_1, \tau_2]$. Note that if $\tau \in \{\tau_1, \tau_2\}$, $q_1 = q_2 \in \{0, 1\}$.

Replace p(T;q) and p(F;q) by the adequate expression to find the range q_1, q_2 .

Corollary 7.

- (i) For any $\tau \leq \gamma \delta$, if $q \geq \hat{q}(\tau)$, $z_+(q, \mathbf{z}_{-i}; \tau) = z_-(q, \mathbf{z}_{-i}; \tau) = 1$ is the only best response for any (non symmetric) vector of seeds $-i \neq i$'s actions.
- (ii) For any $\tau \ge (1 \gamma)d$, if $q \le q(\tau)$, $z_+(q, \mathbf{z}_{-\mathbf{i}}; \tau) = z_-(q, \mathbf{z}_{-\mathbf{i}}; \tau) = 0$ is the only best response for any (non symmetric) vector of seeds $-i \ne i$'s actions.

Proof. Again, it is enough to recall that the number of likes is decreasing in the probability for another seed to share \Box

Corollary 8. Define z_{ps} as the restriction of z to pure strategies. For any (q, τ) , if $z_{ps}^*(q; \tau)$ exists, it is unique.



Figure 15: Illustration of $z_{ps}^{*}(q;\tau)$ with $b=0.2,\gamma=0.75,d=5$

Proof. Consider the parameter space (τ, q) . Proposition 7 describes three subsets of best-responses that do not intersect. No other pure strategies is sustainable, as, by proposition 7, (0,1) is never a best-response.

Figure 15 illustrates the different region of pure strategy best-responses in space (τ, q) . First, one can notice that for some values of τ , the investment of the producer has no effect on the sharing decision of seeds. If τ is *too* low, seeds are not very demanding in terms of likes, so that they are always willing to share. If τ is *too* high, seeds are too demanding in terms of likes, and they never share any information.



Figure 16: Illustration of $z^*(q; \tau)$ in $\tau = \frac{\tau_1 + \tau_2}{2}$ with $b = 0.2, \gamma = 0.75, d = 5$

For intermediate values of τ , however, the symmetric best-response of seeds is fairly similar

to that studied in the benchmark model. To understand so, let us fix a particular value for τ ; we want to understand z^* as a function of q. This means fixing one value of τ on Figure 15 and translating the different areas in term of z. This results in Figure 16, which illustrates the symmetric best-response $z^*(q)$ for $\tau = \frac{\tau_1 + \tau_2}{2}$. Notice that, for this particular τ , $q_1 = 1 - \gamma$ and $q_2 = \gamma$. It means that, for q between $1 - \gamma$ and γ , the symmetric best-response of attention seeking seeds exactly corresponds to that of naive seeds in the benchmark model.

However, for $q \notin [1-\gamma, \gamma]$, attention-seekers' best response changes. Say the producer invests exactly $1-\gamma$. In the benchmark model, upon receiving a positive private signal, seeds were indifferent between sharing or not, as the probability the news was true in such a case was exactly one half. But now, attention seekers' strategies are substitutes; therefore, upon receiving a positive private signal, they can be indifferent between sharing or not only for one particular sharing strategies of the other seeds. This latter strategy is the unique symmetric best-response to q. For $q\left(\frac{\tau_1+\tau_2}{2}\right) < q < 1-\gamma$, z_+^* is strictly increasing in q;²⁸ for $\hat{q}\left(\frac{\tau_1+\tau_2}{2}\right) > q > \gamma$, z_-^* is strictly increasing in q.²⁹

The best-response of attention-seeking seed is thus fairly similar to that of naive seeds for the right value of τ . The problem of seeds as studied in the main text can thus be thought of as a simplification of more complex preferences.

B.3 Other Extensions

B.3.1 Different Setup: Irregular Networks and Seeds' Selection

Denote $\Delta V(d_j)$ the producer's incentive to invest in a regular network of degree d_j as derived in the main text. Let $\Delta V(\delta)$ be the producer's incentive to invest in a network with degree distribution δ . $\Delta V(d_j)$ is continuous in d_j ; hence, there exists a representative degree \tilde{d} such that $\Delta V(\tilde{d}) = \sum_{d_j} \delta(d_j) \Delta V(d_j)$. The equilibria can be characterized applying Proposition 1 and 3 with $d = \tilde{d}$. The role of private knowledge is qualitatively the same for every $\Delta V(d_j)$, hence for $\Delta V(\delta)$: Proposition 2 and Corollary 2 apply. The role of connectivity on the producer's incentive to invest can be assessed in terms of \tilde{d} . The effects of competition through connectivity also carry through as $\Delta V_M(d_j) - \Delta V_D(d_j)$, is continuous in d_j ; hence, there exists a representative degree \check{d} such that $\Delta V_M(\check{d}) - \Delta V_M(\check{d}) = \sum_{d_j} \delta(d_j) (\Delta V_M(d_j) - \Delta V_D V(d_j))$. All other results directly apply.

In addition, from Lemma 2, $\Delta V(d_j)$ is increasing then decreasing; it also concave then $2^{28}z_{+}^{*}$ is implicitly determined by:

$$\frac{\gamma q}{(1-\gamma)(1-q)} = -\frac{\frac{\tau_1 + \tau_2}{2} \frac{b}{1-b} - \frac{1-(1-b(1-\gamma)z_+)^d}{z_+}}{\frac{\tau_1 + \tau_2}{2} \frac{b}{1-b} - \frac{1-(1-b\gamma z_+)^d}{z_+}}$$

 $^{29}z_{-}^{*}$ is implicitly determined by:

$$\frac{(1-\gamma)q}{\gamma(1-q)} = -\frac{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b(1-\gamma)-b\gamma z_-)^d}{1+\frac{\gamma}{1-\gamma}z_-}}{\frac{\tau_1+\tau_2}{2}\frac{b}{1-b} - \frac{1-(1-b\gamma-b(1-\gamma)z_-)^d}{1+\frac{1-\gamma}{2}z_-}}$$

convex. Therefore, it is easy to show that for any distributions of d_j such that $\mathbb{E}(d)$ is before the inflexion point, $\Delta V(\mathbb{E}(d_j)) > \mathbb{E}(\Delta V(d_j))$, indicating that heterogenous degrees are detrimental to investment.

B.3.2 Different Objectives

Behavioral Biases and Partisanship

Consider confirmation bias. When S = -, with probability ϵ , seeds misinterpret the news content and believe it corresponds to their private signal. Then, the probability for an article to be shared becomes: $p_T = \frac{b}{K} \Big[(\gamma + (1 - \gamma)\epsilon) z_+ + (1 - \gamma)(1 - \epsilon) z_- \Big]$ and $p_F = \frac{b}{K} \Big[(\gamma \epsilon + (1 - \gamma)) z_+ + \gamma(1 - \epsilon) z_- \Big]$. The analysis would then be directly applicable. For instance, take a monopoly. $\frac{\partial \Delta V(z)}{\partial \epsilon} = -d(z_+ - z_-) (\gamma(1 - p_F)^{d-1} - 1 - \gamma(1 - p_T)^{d-1}) \le 0$ as $z_+ - z_- \ge 0$, $\gamma < 1 - \gamma$ and $(1 - p_F)^{d-1} \le (1 - p_T)^{d-1}$. The same applies to the duopoly case.

Consider sensationalism. Seeds are assumed to enjoy sharing an article that is not congruent with their private signal because of their taste for sensationalism. In particular, assume that they get a utility premium from such a share of ϵ . Their payoff from sharing is then:

$$u(\text{sharing article } n|\omega = w, S) = \begin{cases} 1 + \epsilon \mathbb{1}_{S=-} & \text{if } n = w \\ -1 + \epsilon \mathbb{1}_{S=-} & \text{otherwise} \end{cases}$$

It follows that their expected utility from sharing when $S = -is 2p(n, s) - 1 + \epsilon$, so that $z_{-|n} > 0$ if $q > \tilde{t}_n = \bar{t}_n = \frac{(1-\epsilon)\gamma \operatorname{Pr}(\omega \neq n)}{(1-\epsilon)\gamma \operatorname{Pr}(\omega \neq n) + \epsilon(1-\gamma) \operatorname{Pr}(\omega = n)}$. Therefore, the news quality is now bounded by \tilde{t}_1 and $\tilde{t}_1 < \bar{t}_1$.

Beyond Visibility

Consider that news quality affects reputation benefits continuously. In particular, assume that the producers revenues can be written $\mathbb{E}(R_k|q) + \nu q$. Then the best-response of the producer would be: $\tilde{q}^*(z) = c^{-1}(\Delta V(z) + \nu) > c^{-1}(\Delta V(z)) = q^*(z)$. However, to understand whether news quality could surpass \bar{t}_1 , one needs to understand whether the producers' best-response might lie completely above the seeds' best-response. This would occur if $c^{-1}(\nu) = c^{-1}(\Delta V((1,1)) + \nu) > \bar{t}_1$. When this is the case, the equilibrium news quality is $c^{-1}(\nu)$; otherwise, one can apply Proposition 1 and 3 with $\tilde{q}^*(z) = c^{-1}(\Delta V(z) + \nu)$.

Consider that news quality affects reputation benefits discretely. In particular, assume that the producers revenues can be written $\mathbb{E}(R_k|q) + \nu \mathbb{1}_{q > \bar{q}}$. Then the best-response of neither side of the market would be affected. However, if $q^* < \bar{q}$, the producer would invest \bar{q} iff $\mathbb{E}(R_k|\bar{q}) + \nu - C(\bar{q}) > \mathbb{E}(R_k|q^*) + \nu - C(q^*)$.